TOTALLY ANTIMAGIC TOTAL LABELING OF COMPLETE BIPARTITE GRAPHS

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ABSTRACT. For a graph $G = (V, E)$ of order $|V(G)|$ and size $|E(G)|$ a bijection from the union of the vertex set and the edge set of G into the set $\{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called a total labeling of G. The vertex-weight of a vertex under a total labeling is the sum of the label of the vertex and the labels of all edges incident with that vertex. The edge-weight of an edge is the sum of the label of the edge and the labels of the end vertices of that edge. A total labeling is called edge-antimagic (respectively, vertex-antimagic) if all edge-weights (respectively, vertex-weights) are pairwise distinct. If a total labeling is simultaneously edge-antimagic and vertex-antimagic at the same time, then it is called a totally antimagic total labeling.
In this paper we prove that complete bipartite graphs admit totally antimagic total labeling.

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1. Introduction

In this paper we consider finite, simple and undirected graphs. In 1990, Hartsfield and Ringel [6] introduced the notion of an antimagic labeling of graph. A graph with q edges is called antimagic if its edges can be labeled with $1, 2, \ldots, q$ without repetition, such that the sums of the labels of the edges incident to each vertex are distinct. They conjectured that every tree except P_2 is antimagic and moreover, every connected graph except P_2 is antimagic. This conjecture was proved true, for all graphs having minimum degree Ω (log $|V(G)|$) by Alon, etc in [1], for more results about antimagic labeling on graphs see [5]. If G is a graph, then $V(G)$ is the vertex set and $E(G)$ is an edge set of G, respectively. A bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called a total labeling of G. A total labeling is called edge-antimagic, if the edge-weights are all distinct. A total labeling is called vertex-antimagic, if the vertex-weights are all distinct. The notion of edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller in [8] as a natural extension of magic valuation defined by Kotzing and Rosa in [7]. Simanjuntak, Bertault and Miller [8] proved that $C_n, C_{2n}, C_{2n+1}, P_{2n}$ and P_{2n+1} have edge-antimagic total labeling. And the notion of vertex-antimagic total labeling of graphs was introduced by Bača, etc in [2], were they proved that paths, cycles and other graphs have vertex-antimagic total labeling. If a graph G with p vertices and q edges possessing a labeling that is simultaneously edge-antimagic total labeling and vertex-antimagic total labeling, then this labeling is called a totally antimagic total labeling, and a graph that admits such a labeling is called totally antimagic total graph. The concept of totally antimagic total labeling was introduced by Bača, etc in [3], were they proved that paths, cycles, stars, double-stars and wheels are totally antimagic total. This concept was introduced as natural extension of the concept of totally magic labeling defined by Exoo, etc in [4], were they proved that K_1, K_3, P_3, cycle C_3 and complete bipartite graph $K_{1,2}$ are the only graphs admits totally magic labeling.

2. MAIN RESULTS

Theorem 2.1. *The complete bipartite graph $K_{n,n}$, admits totally antimagic total labeling, for every $n \geq 3$.*

Proof. Let the vertex set and the edge set of $K_{n,n}$, $n \geq 3$ be

$$V(K_{n,n}) = V_1 \cup V_2 = \{v_i : i = 1, 2, \ldots, n\} \cup \{u_j : j = 1, 2, \ldots, n\},$$
$$E(K_{n,n}) = \{v_i u_j : i = 1, 2, \ldots, n, j = 1, 2, \ldots, n\}.$$

For $n \geq 3$, we define a bijection $f : V(K_{n,n}) \cup E(K_{n,n}) \rightarrow \{1, 2, \ldots, n^2 + 2n\}$ such that
Case 1: if n is even,

$$f(v_i) = \begin{cases} i(n+1) - n & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ i(n+1) & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, \end{cases}$$

$$f(u_j) = \frac{n(n+1)}{2} + j \qquad \text{for } j = 1, 2, \ldots, n,$$

$$f(v_i u_j) = \begin{cases} i(n+1) - n + j & \text{for } i = 1, 2, \ldots, \frac{n}{2}, j = 1, 2, \ldots, n, \\ i(n+1) + j & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, j = 1, 2, \ldots, n. \end{cases}$$

For the edge-weights for $j = 1, 2, \ldots, n$, we get

$$\begin{aligned} wt_f(v_i u_j) &= f(v_i) + f(u_j) + f(v_i u_j) \\ &= \begin{cases} i(n+1) - n + \frac{n(n+1)}{2} + j + i(n+1) - n + j & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ i(n+1) + \frac{n(n+1)}{2} + j + i(n+1) + j & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, \end{cases} \\ &= \begin{cases} \frac{n^2-3n+4ni+4i+4j}{2} & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ \frac{n^2+4ni+n+4i+4j}{2} & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n. \end{cases} \end{aligned}$$

Thus the edge-weights are all distinct, and it easy to observe that edge-weights form the square matrix $A = (a_{ij})_{n \times n}$, where

$$a_{ij} = \frac{n^2-3n+4ni+4i+4j}{2} \quad \text{for } i = 1, 2, \ldots, \frac{n}{2}, j = 1, 2, \ldots, n,$$
$$a_{ij} = \frac{n^2+4ni+n+4i+4j}{2} \quad \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, j = 1, 2, \ldots, n.$$

Hence A is

$$A = \begin{bmatrix} \frac{n^2+n+8}{2} & \frac{n^2+n+12}{2} & \frac{n^2+n+16}{2} & \cdots & \frac{n^2+5n}{2} & \frac{n^2+5n+4}{2} \\ \frac{n^2+5n+12}{2} & \frac{n^2+5n+16}{2} & \frac{n^2+5n+20}{2} & \cdots & \frac{n^2+9n+4}{2} & \frac{n^2+9n+8}{2} \\ \vdots & & & & & \vdots \\ \frac{5n^2+n}{2} & \frac{5n^2+n+4}{2} & \frac{5n^2+n+8}{2} & \cdots & \frac{5n^2+5n-8}{2} & \frac{5n^2+5n-4}{2} \\ \frac{5n^2+5n+4}{2} & \frac{5n^2+5n+8}{2} & \frac{5n^2+5n+12}{2} & \cdots & \frac{5n^2+9n-4}{2} & \frac{5n^2+9n}{2} \end{bmatrix}.$$

From the matrix A it is easy to see that edge-weights are all distinct. For vertex-weights we have the following. First for the set of vertices in V_1, when $i = 1, 2, \ldots, n, j = 1, 2, \ldots, n$, we get

$$\begin{aligned}
wt_f(v_i) &= f(v_i) + \sum_{u_j \in V_2} f(v_i u_j) \\
&= \begin{cases} i(n+1) - n + \sum_{j=1}^{n} f(v_i u_j) & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ i(n+1) + \sum_{j=1}^{n} f(v_i u_j) & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, \end{cases} \\
&= \begin{cases} i(n+1) - n + \sum_{j=1}^{n} (i(n+1) - n + j) & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ i(n+1) + \sum_{j=1}^{n} (i(n+1) + j) & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n, \end{cases} \\
&= \begin{cases} \frac{2i(n^2+2n+1) - n(n+1)}{2} & \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ \frac{2i(n^2+2n+1) + n(n+1)}{2} & \text{for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n. \end{cases}
\end{aligned}$$

It is easy to show that $wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$. Second for vertex-weights of set of vertices V_2, we get

$$\begin{aligned}
wt_f(u_j) &= f(u_j) + \sum_{v_i \in V_1} f(u_j v_i) = f(u_j) + \sum_{i=1}^{n} f(u_j v_i) \\
&= \tfrac{n(n+1)}{2} + j + \sum_{i=1}^{\frac{n}{2}} (i(n+1) - n + j) + \sum_{i=\frac{n}{2}+1}^{n} (i(n+1) + j) \\
&= \tfrac{n(n^2+2n+2)}{2} + (n+1)j \quad \text{for } j = 1, 2, \ldots, n.
\end{aligned}$$

So that $wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_n)$.
Finally, we want to show that the sets of the vertex-weights of vertices V_1 and V_2 do not overlap.
For $i = \frac{n}{2}$, we have

$$wt_f(v_{\frac{n}{2}}) = \tfrac{2(\frac{n}{2})(n^2+2n+1) - n(n+1)}{2} = \tfrac{n^3+n^2}{2} < \tfrac{n^3+2n^2+4n+2}{2} = wt_f(u_1).$$

On the other hand

$$wt_f(u_n) = \tfrac{n^3+4n^2+4n}{2} < \tfrac{n^3+5n^2+6n+2}{2} = wt_f(v_{\frac{n}{2}+1}).$$

So that

$$\begin{aligned}
wt_f(v_1) &< wt_f(v_2) < \cdots < wt_f(v_{\frac{n}{2}}) < wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_n) \\
&< wt_f(v_{\frac{n+2}{2}}) < wt_f(v_{\frac{n+4}{2}}) < \cdots < wt_f(v_n).
\end{aligned}$$

Hence, vertex-weights are all distinct.

Case 2: if n is odd,

$$\begin{aligned}
f(v_i) &= i(n+1) - n & &\text{for } i = 1, 2, \ldots, n, \\
f(u_j) &= n(n+1) + j & &\text{for } j = 1, 2, \ldots, n, \\
f(v_i u_j) &= i(n+1) - n + j & &\text{for } i = 1, 2, \ldots, n, j = 1, 2, \ldots, n.
\end{aligned}$$

For the edge-weights we have

$$\begin{aligned} wt_f(v_iu_j) &= f(v_i) + f(u_j) + f(v_iu_j) \\ &= i(n+1) - n + n(n+1) + j + i(n+1) - n + j \\ &= n^2 - n + 2i(n+1) + 2j \quad \text{for } i = 1,2,\dots,n, j = 1,2,\dots,n. \end{aligned}$$

It is easy to see that the edge-weights are all distinct.
For the vertex-weights we have the following. First for the set of vertices in V_1 we get,

$$\begin{aligned} wt_f(v_i) &= f(v_i) + \sum_{u_j \in V_2} f(v_iu_j) = i(n+1) - n + \sum_{j=1}^{n} f(v_iu_j) \\ &= i(n+1) - n + \sum_{j=1}^{n} [i(n+1) - n + j] \\ &= \tfrac{2i(n^2+2n+1)-n(n+1)}{2} \quad \text{for } i = 1,2,\dots,n. \end{aligned}$$

It is easy to show that $wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$.
Second for vertex-weights of the set of vertices in V_2, we get

$$\begin{aligned} wt_f(u_j) &= f(u_j) + \sum_{v_i \in V_1} f(u_jv_i) = n(n+1) + j + \sum_{i=1}^{n} f(u_jv_i) \\ &= n(n+1) + j + \sum_{i=1}^{n} [i(n+1) - n + j] \\ &= \tfrac{n^3+2n^2+3n+2j(n+1)}{2} \quad \text{for } j = 1,2,\dots,n. \end{aligned}$$

So that $wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_n)$.
Finally, we want to show that the sets of the vertex-weights of vertices V_1 and V_2 do not overlap.
For $i = \frac{n+1}{2}$, we have

$$wt_f(v_{\frac{n+1}{2}}) = \frac{2(\frac{n+1}{2})(n^2+2n+1)-n(n+1)}{2} = \tfrac{n^3+2n^2+2n+1}{2} < \tfrac{n^3+2n^2+5n+2}{2} = wt_f(u_1).$$

On the other hand

$$wt_f(u_n) = \tfrac{n^3+4n^2+5n}{2} < \tfrac{n^3+4n^2+6n+3}{2} = wt_f(v_{\frac{n+1}{2}+1}).$$

So that

$$\begin{aligned} wt_f(v_1) <& wt_f(v_2) < \cdots < wt_f(v_{\frac{n+1}{2}}) < wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_n) \\ <& wt_f(v_{\frac{n+1}{2}+1}) < wt_f(v_{\frac{n+1}{2}+2}) < \cdots < wt_f(v_n). \end{aligned}$$

Hence, vertex-weights are all distinct, this concludes the proof. □

Theorem 2.2. *The complete bipartite graph $K_{n,m}$, $n \leq m/2$ admits totally antimagic total labeling for every $n \geq 3$.*

Proof. Let the vertex set and the edge set of $K_{n,m}$, $n \geq 3$ be

$$\begin{aligned} V(K_{n,m}) &= V_1 \cup V_2 = \{v_i : i = 1,2,\dots,n\} \cup \{u_j : j = 1,2,\dots,m\}, \\ E(K_{n,m}) &= \{v_iu_j : i = 1,2,\dots,n, j = 1,2,\dots,m\}. \end{aligned}$$

For $n \geq 3$, $n \leq \frac{m}{2}$ we define a bijection $f : V(K_{n,m}) \cup E(K_{n,m}) \to \{1,2,\dots,nm+n+m\}$ such that

Case 1: if n is even,

$$\begin{aligned} f(v_i) &= nm + m + i && \text{for } i = 1, 2, \ldots, n, \\ f(u_j) &= j && \text{for } j = 1, 2, \ldots, m, \\ f(v_i u_j) &= m + nj - n + i && \text{for } i = 1, 2, \ldots, n, j = 1, 2, \ldots, m. \end{aligned}$$

For the edge-weights we get

$$\begin{aligned} wt_f(v_i u_j) &= f(v_i) + f(u_j) + f(v_i u_j) \\ &= (nm + m + i) + j + (m + nj - n + i) \\ &= m(n+2) + j(n+1) - n + 2i \quad \text{for } i = 1, 2, \ldots, n, j = 1, 2, \ldots, m. \end{aligned}$$

It is easy to see that the edge-weights are all distinct.
For vertex-weights we have the following. For the set of vertices in V_1, we get

$$\begin{aligned} wt_f(v_i) &= f(v_i) + \sum_{u_j \in V_2} f(v_i u_j) = f(v_i) + \sum_{j=1}^{m} f(v_i u_j) \\ &= (mn + m + i) + \sum_{j=1}^{m} (m + nj - n + i) \\ &= (mn + m + i) + (m^2 + \tfrac{m^2 n + mn}{2} - mn + im) \\ &= \tfrac{m^2(n+2) + m(n+2) + 2i(m+1)}{2} \quad \text{for } i = 1, 2, \ldots, n. \end{aligned}$$

It is easy to show that $wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$.
Second for vertex-weights of the set of vertices in V_2, we get

$$\begin{aligned} wt_f(u_j) &= f(u_j) + \sum_{v_i \in V_1} f(v_i u_j) = f(u_j) + \sum_{i=1}^{n} f(v_i u_j) \\ &= j + \sum_{i=1}^{n} (m + nj - n + i) \\ &= \tfrac{n^2(2j-1) + n(2m+1) + 2j}{2} \quad \text{for } j = 1, 2, \ldots, m. \end{aligned}$$

So that $wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_m)$.

Finally, we want to show that the sets of the vertex-weights of vertices V_1 and V_2 do not overlap. For $j = m$, we have

$$\begin{aligned} wt_f(u_m) &= \tfrac{n^2(2m-1) + n(2m+1) + 2n}{2} \\ &= \tfrac{2n(nm) + nm + nm + 2m + (n - n^2)}{2} \\ &\leq \tfrac{nm^2 + nm + nm + 2m + (n - n^2)}{2} \quad \text{since } (n \leq \tfrac{m}{2}) \\ &< \tfrac{nm^2 + 2m^2 + nm + 2m + (n - n^2)}{2} \quad \text{since } (n < m) \Rightarrow (n < 2m^2) \\ &< \tfrac{nm^2 + 2m^2 + nm + 2m + (2m + 2)}{2} \quad \text{since } (n - n^2 < 0 < 2m + 2) \\ &= wt_f(v_1). \end{aligned}$$

So that
$wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_m) < wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$.

Case 2: if n is odd,

$$f(v_i) = nm + m + n + 2 - 2i \qquad \text{for } i = 1, 2, \dots, n,$$
$$f(u_j) = j \qquad \text{for } j = 1, 2, \dots, m,$$
$$f(v_iu_j) = \begin{cases} m + nj - n + i & \text{for } i = 1, 2, \dots, n, j = 1, 2, \dots, m-1, \\ m + nm + 2 - 2i & \text{for } i = 1, 2, \dots, \frac{n+1}{2}, j = m, \\ m + nm + 2 - 2i + 2n & \text{for } i = \frac{n+1}{2} + 1, \frac{n+1}{2} + 2, \dots, n, j = m. \end{cases}$$

For the edge-weights we get

$$\begin{aligned} wt_f(v_iu_j) &= f(v_i) + f(u_j) + f(v_iu_j) \quad \text{for } i = 1, 2, \dots, n, j = 1, 2, \dots, m, \\ &= \begin{cases} (nm + m + n + 2 - 2i) + j + (m + nj - n + i) \\ \qquad \text{for } i = 1, 2, \dots, n, j = 1, 2, \dots, m-1, \\ (nm + m + n + 2 - 2i) + j + (m + nm + 2 - 2i) \\ \qquad \text{for } i = 1, 2, \dots, \frac{n+1}{2}, j = m, \\ (nm + m + n + 2 - 2i) + j + (m + nm + 2 - 2i + 2n) \\ \qquad \text{for } i = \frac{n+1}{2} + 1, \frac{n+1}{2} + 2, \dots, n, j = m, \end{cases} \\ &= \begin{cases} m(n+2) + 2 + j(n+1) - i & \text{for } i = 1, 2, \dots, n, j = 1, 2, \dots, m-1, \\ 2m(n+1) + n + 4 + j - 4i & \text{for } i = 1, 2, \dots, \frac{n+1}{2}, j = m, \\ 2m(n+1) + 3n + 4 + j - 4i & \text{for } i = \frac{n+1}{2} + 1, \frac{n+1}{2} + 2, \dots, n, j = m. \end{cases} \end{aligned}$$

It is easy to see that the edge-weights are all distinct. For vertex-weights we have the following. First for the set of vertices in V_1, we get

$$\begin{aligned} wt_f(v_i) &= f(v_i) + \sum_{u_j \in V_2} f(v_iu_j) = f(v_i) + \sum_{j=1}^{m} f(v_iu_j) \\ &= \begin{cases} (nm + m + n + 2 - 2i) + \sum_{j=1}^{m-1} (m + nj - n + i) + (m + nm + 2 - 2i) \\ \qquad \text{for } i = 1, 2, \dots, \frac{(n+1)}{2}, \\ (nm + m + n + 2 - 2i) + \sum_{j=1}^{m-1} (m + nj - n + i) + (m + nm + 2 - 2i + 2n) \\ \qquad \text{for } i = \frac{(n+1)}{2} + 1, \frac{(n+1)}{2} + 2, \dots, n, \end{cases} \\ &= \begin{cases} nm + m + \frac{nm^2}{2} + (m^2 + 2n + mi - 5i + 4 - \frac{nm}{2}) & \text{for } i = 1, 2, \dots, \frac{n+1}{2}, \\ nm + m + \frac{nm^2}{2} + (m^2 + 2n + mi - 5i + 4 - \frac{nm}{2} + 2n) & \text{for } i = \frac{n+1}{2} + 1, \frac{n+1}{2} + 2, \dots, n. \end{cases} \end{aligned}$$

So that $wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$.

Second for vertex-weights of the set of vertices in V_2, we get

$$\begin{aligned} wt_f(u_j) &= f(u_j) + \sum_{v_i \in V_1} f(u_j v_i) = f(u_j) + \sum_{i=1}^{n} f(u_j v_i) \\ &= j + \sum_{i=1}^{n} (m + nj - n + i) \\ &= mn + n^2 j + j + \tfrac{n-n^2}{2} \quad \text{for } j = 1, 2, \ldots, m-1, \\ wt_f(u_m) &= j + \sum_{i=1}^{\frac{n+1}{2}} (m + nm + 2 - 2i) + \sum_{i=\frac{n+1}{2}+1}^{n} (m + nm + 2 - 2i + 2n) \\ &= mn + m + n^2 m. \end{aligned}$$

So that $wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_m)$.
Finally, we want to show the sets of the vertex-weights of vertices V_1 and V_2 do not overlap.
For $j = m$, we have

$$\begin{aligned} wt_f(u_m) &= mn + m + n^2 m = mn + m + n(nm) \\ &\leq mn + m + \tfrac{m}{2}(nm) \quad \text{since } (n \leq \tfrac{m}{2}) \\ &\leq mn + m + \tfrac{nm^2}{2} \\ &< mn + m + \tfrac{nm^2}{2} + (m^2 + 2n + m - 1 - \tfrac{nm}{2}) \\ &= wt_f(v_1). \end{aligned}$$

So that $wt_f(u_1) < wt_f(u_2) < \cdots < wt_f(u_m) < wt_f(v_1) < wt_f(v_2) < \cdots < wt_f(v_n)$. Hence, vertex-weights are all distinct, this cocludes the proof. □

3. CONCLUSION

In this paper we proved that complete bipartite graphs $K_{n,n}, n \geq 3$ and $K_{n,m}, n \leq m/2$ are simultaneously vertex-antimagic total and edge-antimagic total.

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