AN M/M/2 RETRIAL QUEUE WITH BREAKDOWNS AND REPAIRS

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Abstract. Retrial queues have been widely used to model many problems arising in telephone switching systems, telecommunication networks, computer networks and computer systems. The reliability study of retrial queues with server breakdowns and repairs is of great importance because of limited ability of repairs and heavy influence of the breakdowns on the performance measures. This paper deals with analysis of the reliability of an M/M/2 retrial queueing system for which the both servers are subject to active and idle breakdowns. We provide also some numerical illustrations to support the obtained theoretical results.

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1. Introduction

Queueing theory is one of the most powerful analytical tools for modeling logistics and communication system [3, 6, 21]. The essential feature of a retrial queue is that an arriving customer finding all servers busy is obliged to abandon the service area and join a retrial group, called orbit, in order to try its luck again after some random time. For a detailed review of the main results and the literature on this topic, the reader is referred to the monographs of Artalejo and Gomez-Corral (2008),Falin and Templeton (1997) [4, 11].

In recent years, there has been an increasing interest in the investigation of the retrial phenomenon in cellular mobile network, see [5, 9, 19, 23] and the references therein, and in many other telecommunication systems including starlike local area networks [13](1997), wavelength-routed optical networks [26](2005), circuit-switched systems with hybrid fiber-coax architecture [12](1998), and wireless sensor network[25](2011). On the other hand, in most of the queueing literature, the server is assumed to be always available, although this assumption is evidently unrealistic. In fact, queueing systems with server breakdowns are very common in communication systems and manufacturing systems, the machine may break down due to the machine or job related problems. This results in an unavailable time period during until the time when the servers become repaired. Such a system with repairable server has been studied as a queueing model and a reliability model by many authors: Falin (2010) [10], Artalejo (1994) [2], Kumar et al (2002) [16], Li and Zhao (2005) [17], Sherman and Kharoufen (2006) [22], Crawford (2007) [8]. Detailed stochastic analysis of a single server retrial queue with server breakdowns and repairs was performed by Kulkarni and Choi (1990) [14]. With the help of the theory of regenerative processes they obtained the generating functions of the limiting distribution and other characteristics of the queue length process for two different models. In Aissani (1994) [1], a version of the unreliable M/G/1/1 retrial queue with variable service was considered. This approach permitted to study the redundancy problem.

It must be noted that current analytical theory of queueing systems with repairable server is limited due to the complexity of the known results. Indeed, the obtained analytical formulas are difficult to use in practice. For that reason, when studying real systems, it is often necessary to replace the real system (usually complex) by a simpler one for which we have exploitable analytical results. This fact led the authors to develop approximate methods to analyze the server reliability and retrial phenomenon. Some approximation methods for this type of queueing models have been elaborated in recent years. In the paper of Oukid and Aissan (2009)[20] , the inequalities and bounds for certain features of the system were obtained through stochastic comparison techniques. Wang and al (2001) [24] used the supplementary variable approach to derive the explicit expressions of some main reliability indexes. For their part, Li and Wang (2006) [18] studied the unreliable M/G/1 retrial queue with two phase service and feedback. Their investigations were also based on the supplementary variable method. The authors obtained some steady state solutions for performance and reliability measures. In general, multiserver retrial queueing systems are difficult to analyze from a mathematical standpoint. If exact results for the steady state probabilities of reliable systems are given only for the single-server and two-servers cases, for an unreliable model we observe their absence when the number of servers exceeds one. In this work, we present an approximate analysis of some reliability indexes of the M/M/2 retrial queue with active and idle breakdowns. The remainder of the paper is organized as follows. In the next section, we give the model description. Section 3 is devoted to the approximate analysis of the server reliability. We derive approximate expressions for the availability and failure frequency of the servers. Finally, some numerical results are given in Section 4.

2. Model description

We consider an unreliable $M/M/2$ retrial queueing system in which customers arrive according to a Poisson process with rate λ ($\lambda > 0$). The service times are assumed to be exponential with rate μ. We assume that there is no waiting space and therefore if at least one of the servers is idle and not failed, then an arriving customer occupies a server immediately. Otherwise, it may choose to enter the orbit with probability P_I to become a source of secondary calls or leave the system with probability $1 - P_I$. It is also assumed that customer that service is interrupted by an active breakdown, must decide whether to join the retrial orbit with probability P_A or leave the system forever after interruption with probability $1 - P_A$. The general structure of the system is shown in Figure 1.

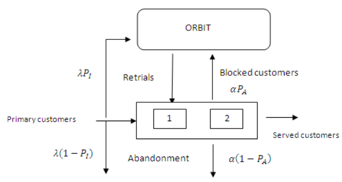

Figure 1. The system structure

Successive inter-retrial times of any customer are independently and exponentially distributed with a common mean $\frac{1}{\nu}$. The input flow of primary arrivals, service times and intervals between repeated attempts are assumed mutually independent. Breakdowns for both servers occur independently via a Poisson process with rate α and the repair times for each server are exponentially distributed with rate β. We assume that each server has its own repairman and repairs begin immediately after a failure. The new random variables and all processes verify the hypothesis of mutual independence.

The state of the system at time t can be described by means of the Markov process $X(t) = \{(N(t), C(t), R(t))\}$, having state space $S = \{(i, j, k) : i \geq 0, j + k \leq 2, j, k \in \{0, 1, 2\}\}$, where

$N(t)$ is the number of customers in the orbit at time t, $C(t)$ is the number of busy servers at time t and $R(t)$ is the number of failed servers at time t.

Let

$$P_{ijk} = \lim_{t \longrightarrow \infty} P(N(t) = i, C(t) = j, R(t) = k); \tag{2.1}$$

$i \geq 0$ and $(j,k) \in E$ with $E = \{(0,0),(0,1),(0,2),(1,0),(1,1),(2,0)\}$ be the steady state distribution of above-mentioned process. Figure 2 depicts the transition diagram for the CTMC.

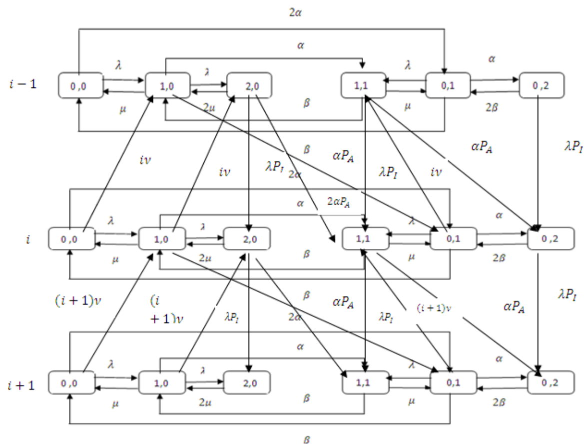

FIGURE 2. transition rate diagram for the unreliable M/M/2 retrial queue

3. RELIABILITY INDEXES OF THE SERVERS

In this section, we provide approximations for the availability and failure frequency of the servers.

Theorem 3.1. *Let $A(t)$ be the probability that the service station is up at time t, which is defined as the point wise availability of the servers, and define the steady-state availability of the server as*

$$A = \lim_{t \longrightarrow \infty} A(t).$$

The approximate availability of the servers is

$$A \approx \frac{\beta^2}{H}[(\beta+\lambda+\mu)(2\mu^2+2\lambda\mu+\lambda^2)+2\alpha(\lambda+\mu)^2]; \tag{3.1}$$

Where the constant H is given by,

$$\begin{aligned} H = {} & \mu^2\alpha^2\lambda + 6\mu^2\alpha^2\beta + 2\mu^3\alpha^2 + 2\mu^2\alpha^3 + 6\beta^2\mu\alpha\lambda + 2\mu\beta^3\lambda \\ & + 4\lambda\beta^2\mu^2 + \lambda^2\beta^3 + 3\lambda^2\beta^2\mu + \lambda^3\beta^2 + 2\alpha\lambda^2\beta^2 + 4\mu^3\alpha\beta \\ & + 6\beta^2\mu^2\alpha + 2\mu^2\beta^3 + 2\mu^3\beta^2 + 2\mu\alpha\lambda^2\beta + 6\mu^2\alpha\lambda\beta + 4\beta\mu\alpha^2\lambda. \end{aligned} \tag{3.2}$$

Proof. A is obtained by considering the following equation

$$A = \sum_{i=0}^{\infty} P_{i00} + \sum_{i=0}^{\infty} P_{i10} + \sum_{i=0}^{\infty} P_{i20}. \tag{3.3}$$

Due to the complexity of obtaining a solution for the state probabilities recursively or by the method of generating functions, we instead resort to an approximate analysis of the system. To determine the approximate probability distributions in the steady state, we apply the phase merging algorithm which was developed in [7] and [15], and proceed in the following manner. In the first time, we find the conditional probability distribution of the number of busy servers at time t, given the number of customers in orbit at time t, then approximate the marginal probability distribution of the number of customers in the orbit.

In order to accurately approximate joint probability distribution of the state of the servers and the number of customers in orbit, we assume that the rates of flow within levels of the orbit are significantly greater than those flowing between levels. Each level is analyzed as a CTMC from which the approximate conditional Probabilities can be found.

1) To find the $P_{l\setminus i}$ we reduce the dimensionality of the state space by defining $Y(t)$ as the state of the servers at time t, such as

$$Y(t) = \{C(t), R(t) : t \geq 0\}; \tag{3.4}$$

Where $C(t)$ and $R(t)$ are defined in (2.1).

Let $P_{l\setminus i} = \lim_{t \longrightarrow \infty} P\{(Y(t) = l \setminus N(t) = i), l = 1, 2, ..., 6\}$ with the index l defined as

state (j,k)	(0,0)	(1,0)	(2,0)	(1,1)	(0,1)	(0,2)
index l	1	2	3	4	5	6

TABLE 1. The susbstitition for server status

This step results in an infinite number of levels which can be analyzed individually. Figure 3 depicts the level for the system.

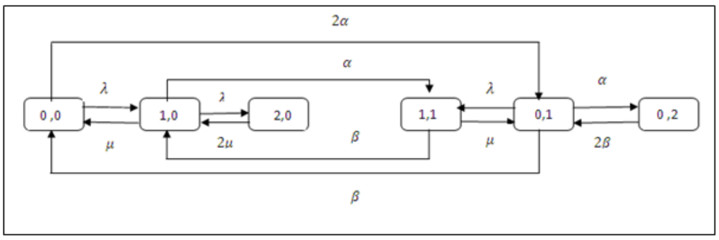

FIGURE 3. the class for the unreliable M/M/2 retrial queue

For each $i \geq 0$, the transition rates of the process $Y(t)$ are described by the following matrix

$$Q_i = \begin{bmatrix} -(\lambda+2\alpha) & \lambda & 0 & 0 & 2\alpha & 0 \\ \mu & -(\lambda+\alpha+\mu) & \lambda & \alpha & 0 & 0 \\ 0 & 2\mu & -2\mu & 0 & 0 & 0 \\ 0 & \beta & 0 & -(\beta+\mu) & \mu & 0 \\ \beta & 0 & 0 & \lambda & -(\lambda+\alpha+\beta) & \alpha \\ 0 & 0 & 0 & 0 & 2\beta & -2\beta \end{bmatrix}$$

Let $p_i = P_{l\setminus i}$

The resolution of the equation $p_iQ_i = 0$ requires solving the following system

$$(3.5)\quad \begin{cases} (\lambda+2\alpha)P_{1\backslash i} = \mu P_{2\backslash i} + \beta P_{5\backslash i} \\ (\lambda+\alpha+\mu)P_{2\backslash i} = \lambda P_{1\backslash i} + 2\mu P_{3\backslash i} + \beta P_{4\backslash i} \\ 2\mu P_{3\backslash i} = \lambda P_{2\backslash i} \\ (\beta+\mu)P_{4\backslash i} = \alpha P_{2\backslash i} + \lambda P_{5\backslash i} \\ (\lambda+\alpha+\beta)P_{5\backslash i} = 2\alpha P_{1\backslash i} + \mu P_{4\backslash i} + 2\beta P_{6\backslash i} \\ 2\beta P_{6\backslash i} = \alpha P_{5\backslash i} \\ \sum_{l=1}^{6} P_{l\backslash i} = 1 \end{cases}$$

Thus we obtain the expressions for the conditional probabilities for all $i \geq 0$. That is

$$(3.6)\quad P_{1\backslash i} = \frac{1}{H}(2(\beta+\lambda+\alpha+\mu)\beta^2\mu^2);$$

$$(3.7)\quad P_{2\backslash i} = \frac{1}{H}(2(\beta+\mu+\lambda+2\alpha)\beta^2\mu\lambda);$$

$$(3.8)\quad P_{3\backslash i} = \frac{1}{H}((\beta+\mu+\lambda+2\alpha)\beta^2\lambda^2);$$

$$(3.9)\quad P_{4\backslash i} = \frac{1}{H}(2(\beta+\lambda+2\mu+2\alpha)\beta\mu\alpha\lambda);$$

$$(3.10)\quad P_{5\backslash i} = \frac{1}{H}(2\alpha\beta\mu^2(\lambda+2\beta+2\mu+2\alpha));$$

$$(3.11)\quad P_{6\backslash i} = \frac{1}{H}(\mu^2\alpha^2(\lambda+2\beta+2\mu+2\alpha)).$$

2) Now each level is considered as a state of a combines CTMC, where the transition rates between levels correspond to customers entering or leaving the orbit. Analysis of this system gives approximate marginal probability distribution of the number of customers in the orbit.

Transition rates between grouped states can be expressed by

$$(3.12)\quad q_{ij} = \begin{cases} \alpha P_A P_{2\backslash i} + (\lambda P_I + 2\alpha P_A)P_{3\backslash i} + (\lambda P_I + \alpha P_A)P_{4\backslash i} + \lambda P_I P_{6\backslash i}, i \geq 0, j = i+1; \\ i\nu(P_{1\backslash i} + P_{2\backslash i} + P_{5\backslash i}), i \geq 1, j = i-1; \\ -[\alpha P_A P_{2\backslash i} + (\lambda P_I + 2\alpha P_A)P_{3\backslash i} + (\lambda P_I + \alpha P_A)P_{4\backslash i} \\ +\lambda P_I P_{6\backslash i} + i\nu(P_{1\backslash i} + P_{2\backslash i} + P_{5\backslash i}], i = j; \\ 0\ elsewhere. \end{cases}$$

Using the substitutions

$$\Lambda = \alpha P_A P_{2\backslash i} + (\lambda P_I + 2\alpha P_A)P_{3\backslash i} + (\lambda P_I + \alpha P_A)P_{4\backslash i} + \lambda P_I P_{6\backslash i};$$

$$\upsilon = i\nu(P_{1\backslash i} + P_{2\backslash i} + P_{5\backslash i});$$

We can see that the analysis of this system is similar to that of the M/M/∞ queueing system. Therefore, we obtain the following marginal probability distribution

$$(3.13)\quad \pi_i = \frac{1}{i!}(\frac{\Lambda}{\upsilon})^i\pi_0, i \geq 0;$$

By using the normalizing equation $\sum_{j=0}^{\infty} \pi_j = 1$ we get

$$(3.14)\quad \pi_0 = e^{-\frac{\Lambda}{\upsilon}}.$$

3) The approximate joint probability distribution of the level of the orbit and state of the servers can be obtained by taking the product of the conditional and marginal probabilities.

$$P_{il} \approx \hat{P_{il}};$$

With

$$\hat{P_{il}} = P_{l\backslash i} \times \pi_i = \frac{P_{l\backslash i}}{i!}(\frac{\Lambda}{\upsilon})^i e^{-\frac{\Lambda}{\upsilon}}. \tag{3.15}$$

With the help of (3.3), the approximate availability of the servers becomes

$$A \approx \sum_{i=0}^{\infty} \hat{P_{i1}} + \sum_{i=0}^{\infty} \hat{P_{i2}} + \sum_{i=0}^{\infty} \hat{P_{i3}}; \tag{3.16}$$

From (3.15), we have

$$\sum_{i=0}^{\infty} \hat{P_{i1}} = \sum_{i=0}^{\infty} P_{1\backslash i} \times \pi_i; \tag{3.17}$$

$$\sum_{i=0}^{\infty} \hat{P_{i2}} = \sum_{i=0}^{\infty} P_2\backslash i \times \pi_i; \tag{3.18}$$

$$\sum_{i=0}^{\infty} \hat{P_{i3}} = \sum_{i=0}^{\infty} P_{3\backslash i} \times \pi_i. \tag{3.19}$$

By substituting (3.6), (3.7), (3.8) in (3.17), (3,18), (3.19) respectively and after calculation of the overall sum, we obtain (3.1). □

Theorem 3.2. *Let F be the failure frequency of the servers. The approximate failure frequency of the servers is given by*

$$F \approx \frac{\mu\alpha}{H}[(\beta + \lambda + 2\mu + 2\alpha)(2\beta\lambda + 2\beta\mu + \mu\alpha) + \beta\mu(2\beta + \alpha)]; \tag{3.20}$$

Where the constant H is given by (3.2).

Proof. F is given by

$$F = \sum_{i=0}^{\infty} P_{i11} + \sum_{i=0}^{\infty} P_{i01} + \sum_{i=0}^{\infty} P_{i02}; \tag{3.21}$$

By using table 1 and (3.15), we find

$$\sum_{i=0}^{\infty} P_{i11} \approx \sum_{i=0}^{\infty} \hat{P_{i4}} = \sum_{i=0}^{\infty} P_{4\backslash i} \times \pi_i; \tag{3.22}$$

$$\sum_{i=0}^{\infty} P_{i01} \approx \sum_{i=0}^{\infty} \hat{P_{i5}} = \sum_{i=0}^{\infty} P_{5\backslash i} \times \pi_i; \tag{3.23}$$

$$\sum_{i=0}^{\infty} P_{i02} \approx \sum_{i=0}^{\infty} \hat{P_{i6}} = \sum_{i=0}^{\infty} P_{6\backslash i} \times \pi_i. \tag{3.24}$$

By substituting (3.9), (3.10), (3.11) in (3.22), (3.23), (3.24) and after some algebra, we obtain (3.20). □

4. Illustrative numerical examples

In the first time, we present numerical illustrations to assess the quality of the phase-merging approximation used to approximate the unreliable M/M/2 retrial queue reliability indexes, in section 4.1. In the second time, we investigate the impact of the parameters on the availability and failure frequency of the servers without taking into account the retrial rate. Since these indexes are independent of this parameter, so we only vary λ, μ, α, β. Recall, that for the algorithm to produce effective results we require that the flows within levels of the orbit be significantly greater than those between levels.

4.1. **Evaluation of the quality of the approximation of the Phase Merging Algorithm.** To evaluate the quality of the approximations that we have obtained, we present two examples.

Example 1. In this example, we fix $\mu = 6$, $\lambda = 4$, $\beta = 0.1$ and vary α from 0 to 2 in increments of 0.01.

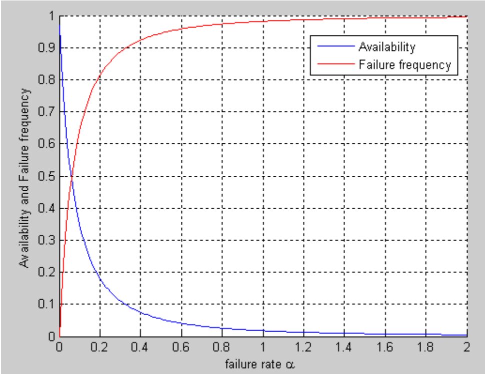

FIGURE 4. Availability and failure frequency vs.α

Figure 4 describe the influence of the parameter α on the both availability and failure frequency of the servers. As is expected (intuitively), with the increase of failure rate α, the probability of finding a server in good condition is almost zero which leads to decrease the availability of servers and increase the failure frequency of servers.

Example 2. In this example, we fix $\mu = 5$, $\alpha = 0.1$, $\lambda = 2$ and vary β from 0 to 1 in increments of 0.01.

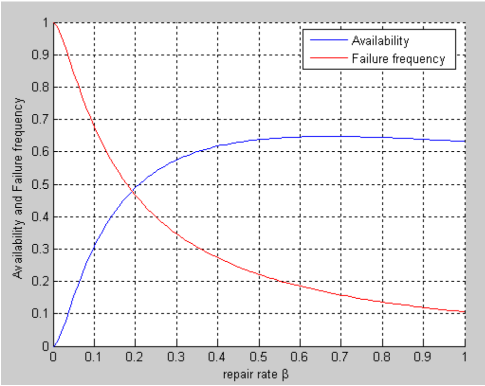

FIGURE 5. Availability and failure frequency vs.β

In figure 5 we describe the variations of availability and failure frequency of the servers in function of repair rate. It is also intuitive that if the service rate is greater than the arrival rate and with the increase of repair rate, the servers will be often available which leads to decrease the failure frequency.

4.2. **Impact of parameters on the calculated approximate reliability indexes.** First, we present an example where the assumption that the flows within levels of the orbit be significantly greater than those between levels is violated. Then we give two examples that show us how to choose the parameters

λ and μ to have a good server availability.

Example 3. We fix $\mu = 10$, $\beta = 5$, $\alpha = 30$ and vary λ from 0 to 200 in increments of 0.1. In this

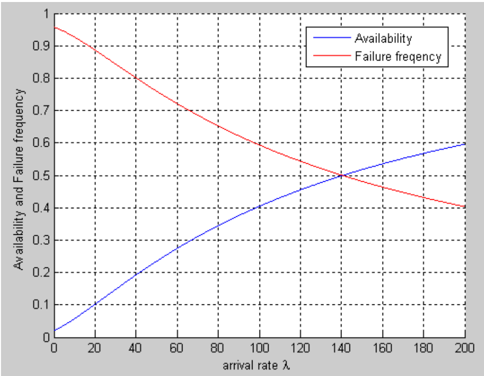

FIGURE 6. Availability and failure frequency vs.λ

example we can see that the assumption is violated. In figure 6, we show that the availability increases rapidly and failure frequency decreases, while the repair rate is very small compared to the failure rate. Moreover, the service rate is small and the arrival rate is increasing. So if the assumptions mentioned above, are violated, the method may perform very poorly.

Example 4. In this example, in the first time we fix $\lambda = 10$, $\alpha = 0.1$, $\beta = 2$ and vary μ from 1 to 200 in increments of 0.1, in the seconde time we put $\lambda = 70$ and keep the same values for other parameters.

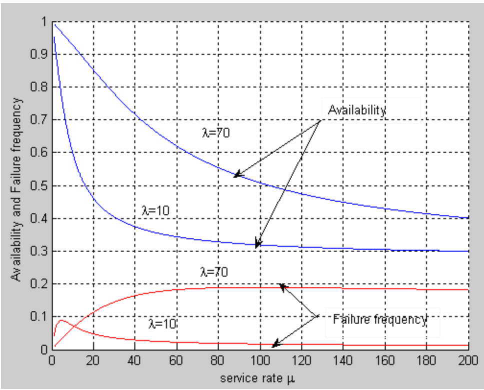

FIGURE 7. Availability and failure frequency vs.μ

Looking at figure 7, we notice that the availability decreases with increasing values of μ, on the other hand the failure frequency knows a little higher then it decreases. However, the availability will be greater when the arrival rate is high, so for a good server availability and smaller failure frequency, the λ and μ parameters must be approaching.

Example 5. Finaly, we fix $\lambda = 4$, $\mu = 6$, $\alpha = 0.1$ and vary β from 0 to 5 in increments of 0.01.

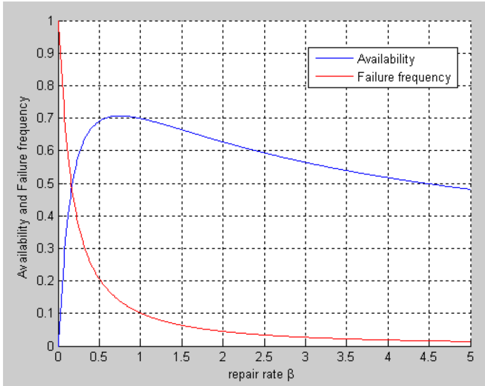

Figure 8. Availability and failure frequency vs.β

Figure 8 describe the influence of the parameter β on the availability and failure frequecy.

We observe that with increasing values of repair rate, the failure frequency decreases and server availability increases (which is very logical). But from a certain rank of repair rates, availability begins to decline, so a small failure rate needs a small repair rate.

5. Conclusion

The M/M/2 retrial queue with active and idle breakdowns has been investigated. Some approximate reliability indexes of the model are obtained by using the approximate state probabilities determined by applying phase merging algorithm. These theoretical investigations are supported by numerical illustrations. The model presented in this article is novel and it is realistic as it can be applicable to various congestion situations encountered in telecommunication systems, computer networks, banks which involve the use of multiservers at the same time.

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