

## TRANSITION FROM 2D TO 3D WITH GEOGEBRA

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**ABSTRACT:** This article presents the definition of projection plane, its importance for the geometry constructions used in civil engineering and comparative analysis of three opportunities for creating a three dimensional basis, used in drawing such a plane. First method consists of transforming affine and orthonormal coordinates and its application in GeoGebra is presented. Second method, using combination of spherical and polar coordinates in space, is introduced. The third suggested method is an application of descriptive geometry for transforming  $2D$  to  $3D$  and a new method of forming a plane of projection, which will be used later in the reviewed example below. The example shows how GeoGebra software can be used in technical drawing used in civil engineering.

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### 1. INTRODUCTION

$3D$  modeling capabilities allow the creation of  $3D$  images that are as realistic as the actual objects (see [1]). These images are called  $3D$  models because, just like a physical model, they can be rotated on the screen. Views from a  $3D$  model, such as isometrics or perspectives, can be displayed from any angle with a few simple steps.

The main aim of the paper is to represent of  $3D$  objects by GeoGebra, which allows rotations, translation and dilations to view things from different angles and to zoom in. This method can be very useful in civil engineering (see [3]). Also GeoGebra can be used in mathematical statistics (see [5]).

### 2. FIRST MODEL OF THREE DIMENSION BASIS CONSTRUCTION

M. Passante has developed the following construction (see [10]). The  $3D$  rotation matrices for the  $x$ ,  $y$  and  $z$  axis are

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \quad R_y = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are angles between  $0^\circ$  and  $360^\circ$ . The composition of the rotations  $R = R_x R_y R_z$  is not commutative.

After rotation  $R$  the point  $P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$  will have 3D coordinate  $RP = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix}$ , where

the  $q$  coordinates depend on the angles  $\alpha$ ,  $\beta$  and  $\gamma$ . To see what this looks like in 2D (on the screen) it is necessary to use two of the coordinates of  $RP$ . If the  $x$  axis pointing out of the screen towards the viewer then the screen coordinates are  $y, z$

which means to plot the point  $Q = \begin{pmatrix} q_y \\ q_z \end{pmatrix}$ .

This construction is good, but it is not very flexible. As well as  $Q$  it will be necessary to extract some other information from the 3D point  $RP$ , and it is not easy to do in GeoGebra with  $RP$  in this form. Instead it is possible to start with

$$P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = p_x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + p_y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + p_z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = p_x E_x + p_y E_y + p_z E_z.$$

Thus  $RP = p_x RE_x + p_y RE_y + p_z RE_z$ . Then  $p_x, p_y, p_z$  are components of the rotated  $P$  in the directions of the rotated axis  $RE_x, RE_y, RE_z$ . If  $W_x$  is the 2D vector with components the  $y$  and  $z$ , coordinates of  $RE_x$ , and similarly for  $W_y$  and  $W_z$ , then  $Q = p_x W_x + p_y W_y + p_z W_z$ .

The vectors  $p_x W_x, p_y W_y, p_z W_z$  are the components of the rotated  $P$  along the rotated axis as viewed on the Fig. 1.

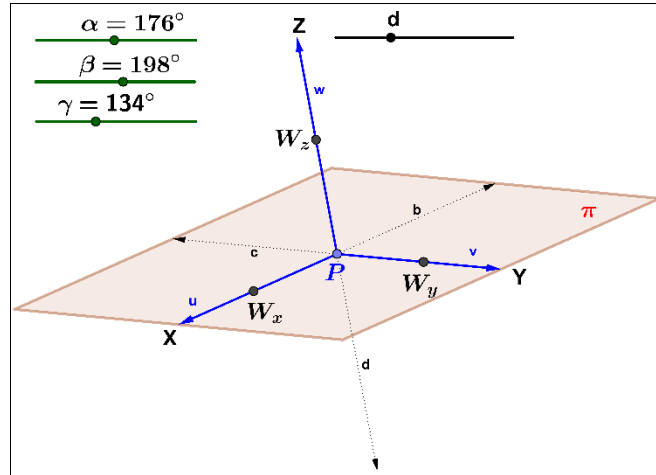


Fig. 1 New axes with constructed plane

Stepwise construction of the plane is presented in Table 1.

Table 1

Command/Tool	Corresponding construction
$a = 2$	Constructing sliders $a$ , $b$ , $c$ from $0^\circ$ to $360^\circ$ , $d$ – from 0.5 to 5 in steps of 0.1 which will be used to lengthen and shorten axes (Fig. 1).
$E_x = \{\{1\}, \{0\}, \{0\}\}$ , $E_y = \{\{1\}, \{0\}, \{0\}\}$ , $E_z = \{\{1\}, \{0\}, \{0\}\}$	This represents the column vectors $E_x, E_y, E_z$ .
$R_x = \{\{1, 0, 0\}, \{0, \cos(a), -\sin(a)\}, \{0, \sin(a), \cos(a)\}\}$ $R_y = \{\{\cos(b), 0, -\sin(b)\}, \{0, 1, 0\}, \{\sin(b), 0, \cos(b)\}\}$ $R_z = \{\{\cos(c), -\sin(c), 0\}, \{\sin(c), \cos(c), 0\}, \{0, 0, 1\}\}$ $R = R_x * R_y * R_z$	Rotation matrices $R_x, R_y, R_z$ and the dilation matrix $R$ .
$V_x = R * E_x$ , $V_y = R * E_y$ , $V_z = R * E_z$	Rotation and dilation of the unit vectors $E_x, E_y, E_z$ .
$W_x = (\text{Element}[\text{Element}[V_x, 2], 1], \text{Element}[\text{Element}[V_x, 3], 1])$ $W_y = (\text{Element}[\text{Element}[V_y, 2], 1], \text{Element}[\text{Element}[V_y, 3], 1])$	Creating 2D points to be viewed on the screen by extracting 2 <sup>nd</sup> and 3 <sup>rd</sup> components of the

$W_x = (\text{Element}[\text{Element}[V_z, 2], 1], \text{Element}[\text{Element}[V_z, 3], 1])$	column vectors $V_x, V_y, V_z$
$u = \text{Vector}[d*W_x], v = \text{Vector}[d*W_y], w = \text{Vector}[d*W_z]$	Constructing axes.
$b = -u$ $c = -v$ $d = -w$	Constructing the opposite vectors.
	Hiding the points $W_x, W_y, W_z$ .
	In the Starting Point Box in Properties and Position menu typing the following $X = 2*W_x;$ $Y = 2*W_y;$ $Z = 2*W_z.$
$\text{Polygon}[d*(W_x+W_y), d*(-W_x+W_y), d*(-W_x-W_y), d*(W_x-W_y)]$	Constructing plane. With right click of each side of this polygon click off Show Object.

### 3. SECOND MODEL OF THREE DIMENSION BASIS CONSTRUCTION

Let  $Oxy$  be an orthogonal coordinate system. The point  $M(x, y)$  is given to it. The values  $\rho = |\overrightarrow{OM}| \geq 0$ ,  $\theta = \angle(Ox, \overrightarrow{OM})$ ,  $\theta \in [0, 2\pi)$  are called polar coordinates (Fig. 2).

$$\begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \end{cases}$$

Let  $Oxyz$  be an orthogonal coordinate system. The point  $M(x, y, z)$  is given to it.

The values  $\rho = |\overrightarrow{OM}|$ ,  $\rho \in [0, \infty)$ ,  $\theta = \angle(Oz, \overrightarrow{OM})$ ,  $\theta \in [0, \pi]$  and  $\varphi = \angle(Ox, \overrightarrow{OM'})$ ,  $\varphi \in [0, 2\pi)$ , where  $M'$  is the orthogonal projection of  $M$  on  $Oxy$  are the spherical coordinates of point  $M$  (Fig. 3).

$$\begin{cases} x = \rho \sin \varphi \cos \theta \\ y = \rho \sin \varphi \sin \theta \\ z = \rho \cos \varphi \end{cases}$$

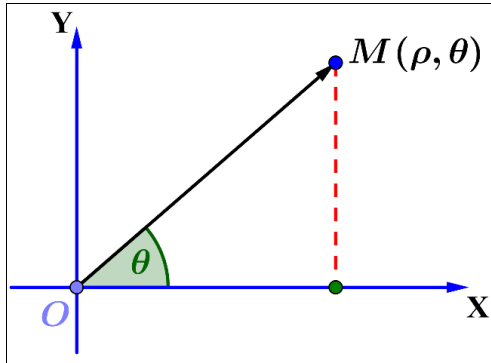


Fig. 2 Polar coordinates

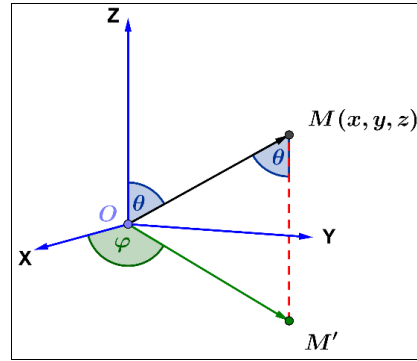


Fig. 3 Spherical coordinates

Using the above are constructed projection points  $U_x, U_y, U_z$  in Table 2, which coordinates are combination of polar and spherical coordinates and vectors connecting them with center  $O$  of the orthogonal coordinate system (see [9]). With help of them is going to be changed coordinate system as refer on Fig. 4.

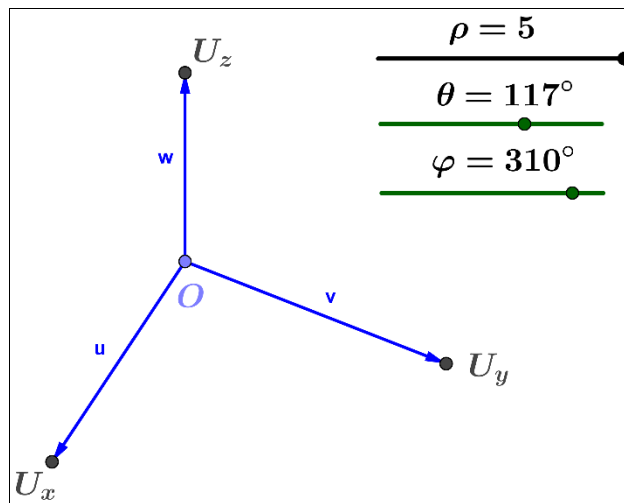


Fig.4 New coordinate system

Table 2

Command/Tool	Corresponding construction
$O = (0,0)$	Intersection of the two axes with point $O$ .
$a = 2$ 	Constructing sliders $\varphi$ from $0^\circ$ to $360^\circ$ , and $\theta$ from $0^\circ$ to $180^\circ$ for

	rotation the axes, $\rho$ - from 0 to 5 in steps of 0.01 which will be used to lengthen and shorten them (Fig. 4).
$U_x = (\rho \sin(\theta), -\rho \cos(\theta) \sin(\varphi))$ $U_y = (\rho \cos(\theta), \rho \sin(\theta) \sin(\varphi))$ $U_z = (0, \rho \cos(\varphi))$	Creating three points which coordinates are combination of polar and spherical coordinates to realize the rotation in space effect.
u, v, w	Constructing axes by connecting center O with points $U_x, U_y, U_z$ via vectors u, v, w.
	Hiding the points $U_x, U_y, U_z$ .

Construction of the plane using translation and rotation is presented in Table 3.

Table 3

Command/Tool	Corresponding construction
View → Graphics 2	Opening a new Graphics window
<b>Graphics 2 window:</b> Polygon[A, A_1, 4]	In the Graphics 2 window is constructed a polygon with several vertices, in Fig. 5 with 4 vertexes.
<b>Graphics window:</b> $A' = \text{Translate}[O, U_x * x(A) + U_y * y(A)]$ $A'_1 = \text{Translate}[O, U_x * x(A_1) + U_y * y(A_1)]$ $A'_2 = \text{Translate}[O, U_x * x(A_2) + U_y * y(A_2)]$ $A'_3 = \text{Translate}[O, U_x * x(A_3) + U_y * y(A_3)]$	Switching back to first Graphics window. Translating center O of the new coordinate system by vectors constructed in the following manner, which connects polygon from Graphics 2 window with rotation of the first window. It is created new points which are dependent on the angles of rotation.
Polygon[A', A'_1, A'_2, A'_3]	Constructing a plane through translated points $A', A'_1, A'_2, A'_3$ . It is positioned now in the base of the rotated coordinate system and is made to look dynamical (Fig. 5).
	With right click of each side of this polygon click off Show Object. Hiding the points $A', A'_1, A'_2, A'_3$ and Graphics 2 window.

It is seen in Fig. 5 that if any of the points  $A, A_1, A_2, A_3$  moves in Graphics 2 window, the corresponding point in Graphics window will also move, but if the rotated view of the plane in the Graphics window is changed it will not change the

rotated view of it in the Graphics 2 window. Now the Graphics 2 window can be hidden for future drawings because it is not necessary but must not be closed.

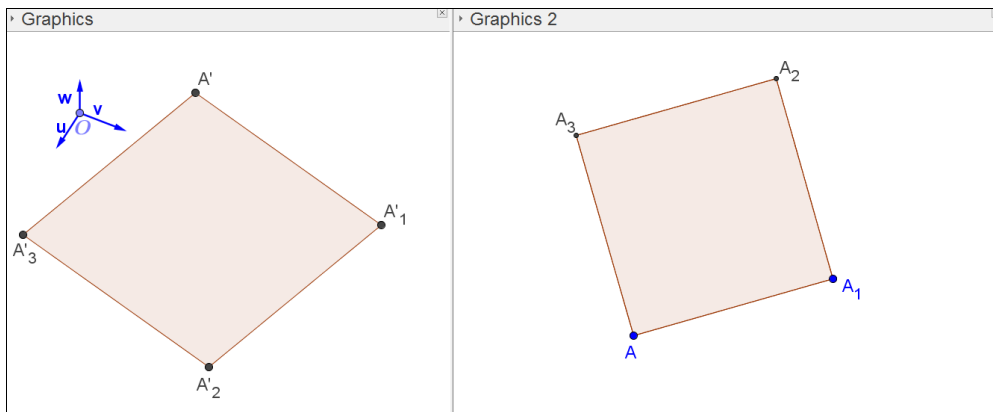


Fig. 5 Plane view in the two Graphics' windows

Methods reviewed above requires good knowledge in theory and steps of constructing are quite difficult despite there are some similarities see ([6], [7]). For this reason is introduced the third method which is based on simple constructions without necessity of previous knowledge of theory and contains a certain amount of flexibility.





#### 4. CONSTRUCTION OF A PROJECTION PLANE

##### 4.1 Orthogonal projection of a given point

V. Dubrovskiy and S. Pozdnyakov (see [2]) have been presented the model of cube construction in the interactive geometric environment. This model will be applied in GeoGebra.

A tool for GeoGebra construction of an orthogonal projection of a given point is presented in the Table 4.

Table 4






Command/Tool	Corresponding constructions
$y = 0$	Constructing a straight line, via the input field.
	Choosing a point A on it.
	Constructing a straight line through A, perpendicular to the straight line.
	Constructing the point of crossing of the two straight lines.
	Hiding the perpendicular straight line.
	Input objects: 1) two points, determining the straight line (or the straight line itself), 2) point A, which will be projected. Output

	objects: projected point A'; Name and icon: <b>Projection</b> ,  .
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It is necessary to construct a dynamic model, allowing the observation of how the projection of a circle is changing on a plane on the screen. After this a polyhedron will be inscribed in the circle.

The circle is positioned on the plane itself on the screen. The projection of the diameter, which is perpendicular to the rotation axis, comes closer while rotating, whereupon the coefficient of approximation  $k$  is equal to the cosine of the angle of rotation (or the angle of slope), which is also the angle between the screen and the plane of the circle. In this regard, the other hordes will come closer, perpendicular to the diameter – the axis of rotation. Therefore, it is enough to construct a model of the circle, approaching its horizontal diameter (Table 5).

Table 5

Command/Tool	Corresponding constructions
$y = 0$	Constructing a horizontal straight line (parallel to the $x$ axis), via the input field.
	Creating a slider $b$ for the scale increasing of the drawing.
	Constructing a circle with a center on the straight line and the radius $b$ .
	Choosing a point $P$ on the circle and project it on the straight line with the help of the <b>Projection</b> tool; Name of the projected point $P'$ (Fig. 5b).
	Constructing a slider for controlling of the approximation coefficient $k$ . In this way $k$ will be able to change between 0 and 1, which meets the angle of the slope from $0^\circ$ to $90^\circ$ .
	Applying a homothetic with a center $P'$ and coefficient $k$ to the point $P$ .

Point  $P$  on the circle, obtained due to homothetic point during the movement, will inscribe a circle close to the axis  $x$  with coefficient  $k$ . Its trajectory can be constructed with the help of the Locus tool, by separating the point  $P'$  and its prototype. The obtained curve is an ellipse. Moving the slider, it is possible to see how the circle is rotating around the axis  $y$  on the screen even though this effect is obtained with the approximation of the circle.

#### 4.2 Construction of a square projection inscribed in a circle

As observation, the projection will perform with the square inscribed in the circle while bending it, as well as the rotation of the square around the center of the circle.

A square will be inscribed in a circle with vertex in point  $P$ , in order for point  $P$  to rotate it successively three times by  $90^\circ$  around the center  $O$  of the circle (or to construct two perpendicular diameters, or to take advantage of the tool Regular polygon).



Next, the four vertices of the square will be obtained via the operation approximation to the diameter with a coefficient  $k$ , repeating the steps, written above (for which a tool can be created also). The points obtained lie on an ellipse (this fact can be used for control of the regular construction). They are connected with segments (Fig. 6). A parallelogram is formed, which consists of a parallel projection of the square, inscribed in the circle, rotated around a horizontal axis.

Therefore, a model of the basis of a polyhedron has been constructed – a square which can:

- ✓ Rotate around a horizontal axis, while moving the slider  $b$ ,
- ✓ Rotate the square around the axis, going through its center  $O$  and perpendicular to its plane when point  $P$  is moved on the circle (Fig. 7). Smaller  $k$  slider.

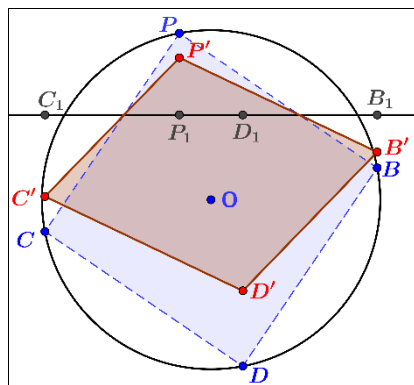


Fig. 6  $k = 0.7$

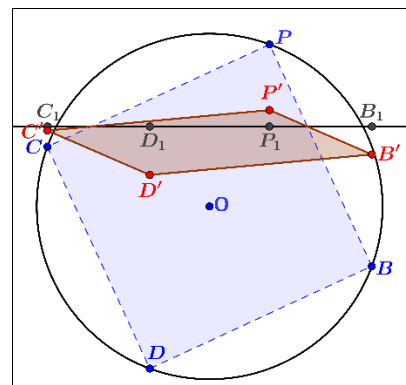


Fig. 7  $k = 0.2$

Analogically, other bases of a polygon can be created – triangle, rectangle, regular hexahedron and others.

### 4.3 Rotating cube model

In order to construct a model of the cube, its basis will be transferred with a vector perpendicular to its plane and equal to the length of its side, thus obtaining its base. Since the vector of the transfer is perpendicular to the horizontal axis, its projection will be perpendicular too. The only thing to be done is to define its length. Therefore, the “approximation” construction will be observed (Fig. 7). On the plane of the screen a vertical straight line will appear, the circle which started the whole construction – segments with lengths  $2R$ , where  $R$  – its radius, and the vector of the transfer – vector  $\overline{OQ}$  perpendicular to this segment, length  $a$  of which is equal to the rib of the cube, then this is the side of the square inscribed in the circle:  $a = R\sqrt{2}$ , if the plane of the circle is bent under an angle  $\alpha$  towards the screen, then  $a \cos \alpha = k$  and the projection of the segment  $OQ$  of the screen – this segment  $OQ$ , the length of which

is equal of  $a \sin \alpha = am$ , where  $m = \sqrt{1-k^2}$ . Now what is left is to use these calculations for the following model.

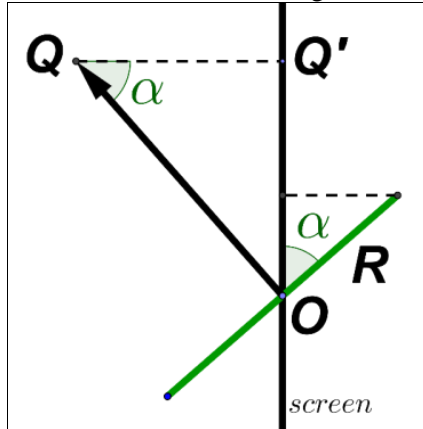


Fig. 8 - Slope of the plane on the screen

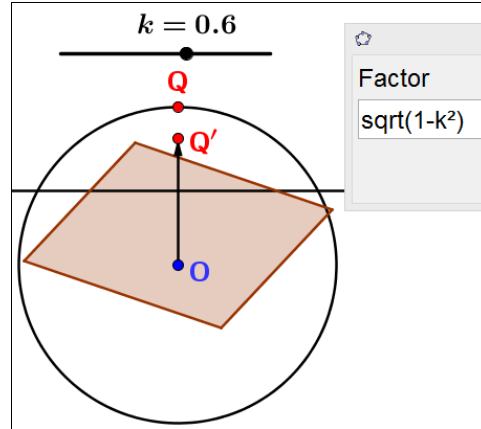


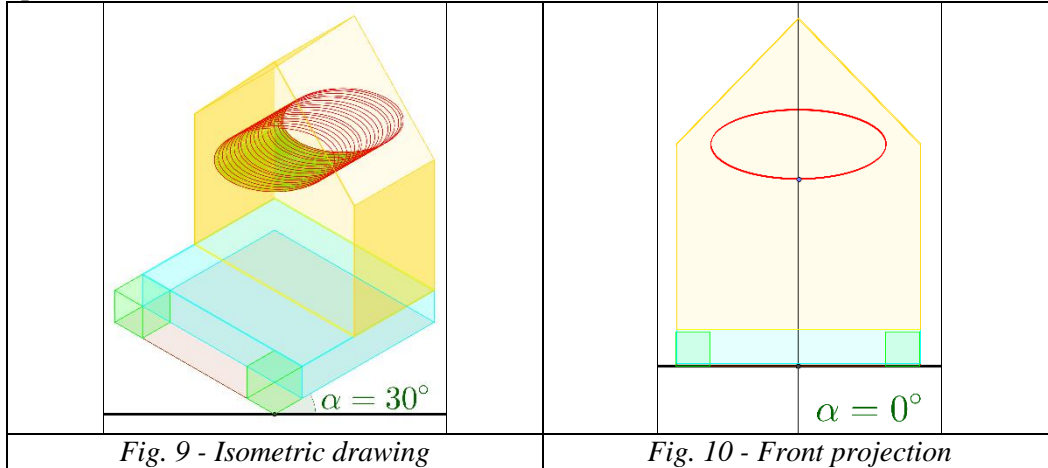
Fig. 9 – Dilation by factor

A vertical radius  $OF$  will be constructed in the output circle. On its extension, the segment  $OQ_0 = R\sqrt{2}$ , equal to the rib of the cube (the side of the cube inscribed in the circle) will be put. In point  $Q_0$ , a homothetic with a coefficient  $\sqrt{1-k^2}$  and a center  $O$  are included. Thus, a point  $Q'$  is obtained. Then - vector  $\overrightarrow{OQ'}$  - the projection of the vector of the transition of the plane on the screen (Fig. 9). It is on this vector that the cube must be transferred in order to obtain its parallel side. Performing this transition and connecting the vertexes of the output square and its images it will receive a cube. All that is left is to hide all additional constructions and to leave only the slider for changes  $k$ , controlling the slope and the output point  $P$  of the circle for rotating around the axis perpendicular to the base of the cube.

## 5. APPLICATION IN TECHNICAL DRAWING USED IN CIVIL ENGINEERING.

The types of three-dimensional representation drawings that are relevant to this article include para-line - isometric and planometric (see [11]). Para-line drawing objects are drawn with the receding lines remaining parallel to each other (hence the term 'para-line'). Common types of para-line drawings include isometric and planometric ones. Isometric drawings are constructed with both sides receding from the corner edge at 30 degrees. The isometric drawing provides a comprehensive overall view of the object. Otherwise planometric drawings are the base (or plan) of the object retains its true form (is not altered) with both sides receding at 45 degrees (or one side recedes at 30 degrees and the other at 60 degrees).

To demonstrate technical drawing from another aspect of view in GeoGebra, a model which is much more than a 2D drawing has been constructed (see [4]). It was made dynamic and scaling in any direction. To put the defined model in isometric (Fig. 9) or planometric para-line, the sliders *rotation* and *slope* have to be set into appropriate parameters (Table 6).



*Table 6*

Figure №	Measurements of sliders
Fig. 9	Scale = 5; rotation = $90^\circ$ ; slope = 0.58; height = 10; length = 1; ratio = 1.
Fig. 10	Scale = 5; rotation = $45^\circ$ ; slope = 0; height = 10; length = 1; ratio = 1.

Sliders in Table 6 and Table 7 are described as follow: scale expands and constricts the whole drawing, rotation – rotation of the figure by angle from  $0^\circ$  to  $360^\circ$ , slope – dilation of the plane on the screen, height – height of the roof up from the blue base, length controls the length of the rib of small cubes, the height of the blue base and changing focuses of the ellipse, ratio is constricting just the width of the blue base, ratio is scaling the width of the roof and opens the projection plane to be seen under the construction.

The advantage of this is that the model can be rotated and seen in different angles and slopes, thus inspecting every element of the Bird house.

Just rotating the model, it can be seen in third angle projection views or any wanted view (Fig.10). For the needs of this project, a Bird house model is made to transform dynamically: the height of the roof, the entire middle construction, and the basis proportions are changeable, so the viewer can see under it the plane of projection  $\alpha$  on which the construction steps (Fig. 11 – 16). The sliders' values are described in Table 7.

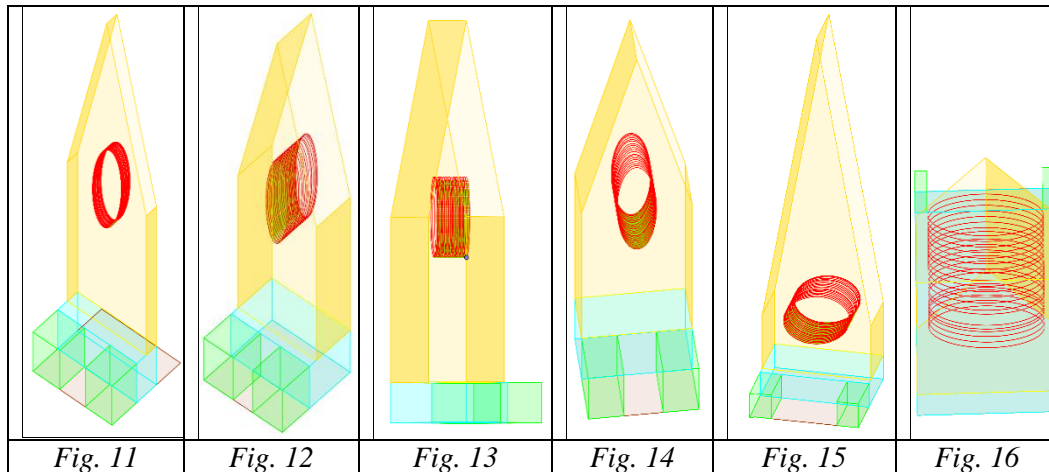


Table 7

Figure №	Measurements of sliders
Fig. 11	Scale = 2.2; rotation = $86^\circ$ ; slope = 0.61; height = 10; length = 0.9; ratio = 0.3.
Fig. 12	Scale = 2.2; rotation = $86^\circ$ ; slope = 0.61; height = 10; length = 1; ratio = 1.
Fig. 13	Scale = 2.2; rotation = $0^\circ$ ; slope = 0; height = 10; length = 1; ratio = 1.
Fig. 14	Scale = 2.2; rotation = $38^\circ$ ; slope = 0.64; height = 10; length = 1; ratio = 1.
Fig. 15	Scale = 2.2; rotation = $58^\circ$ ; slope = 0.41; height = 0.6; length = 1; ratio = 1.
Fig. 16	Scale = 5; rotation = $224^\circ$ ; slope = 0.91; height = 10; length = 0.6; ratio = 1.

## 7. CONCLUSION

It is possible to extend the application in technical drawing in perspective drawings where objects are drawn in a naturalistic manner consistent with human vision; the receding lines converge towards the horizon (eye level) rather than remaining parallel to each other. The placement of the horizontal line determines the location of the

viewer and provides capacity for different views of an object or the relationship of parts to each other (see [8]).

In the above example is demonstrated that the knowledge and understanding of how to draw an ellipse is important to this study. Whilst ellipse templates can be useful, students should know how to draft a freehand ellipse for para-line and perspective purposes.

The use of different line styles and widths is important in technical drawing as they are used to indicate details and features in a drawing. Line styles make drawings easier to read: for example, solid lines used to show the object will stand out from broken lines used to show hidden information. This is very easy to be fulfilled in GeoGebra.

Placement of numeric information, known as dimensioning, measures directly from an orthogonal drawing when the scale is 1:1 (full size). All dimensions are recorded using true size measurements. Where the object does not fit to the page, reduction ratios are used.

The method examined is suitable for dynamic demonstration of technical drawing lectures and gives future engineer designers one more possibility to expand their potential with lateral thinking and new creative methods to design and create their projects.

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