ROMANIAN JOURNAL OF MATHEMATICS AND COMPUTER SCIENCE, 2015, VOLUME 5, ISSUE 1, p.84-94

MODELING OF THERMOELASTIC WAVES IN ROTATING CYLINDRICAL PANEL BY USING MATRIX FROBENIUS METHOD

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ABSTRACT**:** In this paper, free vibrations are investigated in a homogeneous transversely isotropic, rotating cylindrical panel, in context of the linear theory of thermoelasticity. Three displacement potential functions have been introduced in the equations of motion and heat conduction in order to decouple purely shear and longitudinal motions. The purely transverse wave is not affected by thermal field. By using the method of separation of variables, the system of governing partial differential equations is reduced to four second order coupled ordinary differential equations in radial coordinate. The Matrix Frobenius method of extended power series is employed to obtain the solution in radial direction. The secular equations are obtained by using traction free and thermally insulated boundary conditions. In order to illustrate the analytic results, the numerical solution of various relations and equations has been carried out to compute the lowest frequency, phase velocity, frequency shift and damping factor of vibrations in a rotating cylindrical panel of zinc material with MATLAB software programming. The computer simulated results have been presented graphically.

Mathematics Subject Classification (2010): 74H45,74J20, 74K25.
Keywords: Damping; Frequency shift; Frobenius method; cylindrical panel

Article history:
Received 10 May 2015
Received in revised form 8 July 2015
Accepted 9 July 2015

1. INTRODUCTION

As one of the common structural element in many engineering fields such as aerospace, civil, chemical, mechanical, naval and nuclear etc. cylindrical panel has been of interest to large number of researchers. The effect of rotation on cylindrical panel has its own important in the design of high speed steam, gas turbine, rotation rate sensors etc. It is also well known that, when a structure is exposed to high rate of thermal loading, the induced displacement and temperature field cannot analyze independently. In this case, thermo mechanical coupling must be taken into account in analysis. However, the analysis of vibrations characteristics of thermo elastic rotating cylindrical structures is more complex because of the equation of motion together with boundary conditions. The previous collection of works on vibrations of isotropic curved panels was published in [7, 10, 12, 24]. An iterative approach to predict the frequencies of isotropic cylindrical shells and panels based on three-dimensional elasticity was employed by Soldatos and Hadhgeorgiou [20]. Leissa [6] investigated the vibrations of thick cylindrical panel using the Ritz Method.

Thermal stresses and deflections that occurred in a composite cylinder due to a uniform rise in temperature were studied by Hallam and Ollerton [5] and they compared the obtained results by a special application of the frozen stress technique of photo elasticity. The theory of thermoelasticity is well established by Nowacki [11]. MacQuillen and Brull [9] investigated the coupled thermoelasticity of thin shell by using Galerkin Method. They considered the first order shell theory, based on love assumptions, and essentially ignored normal stress, transverse stress and rotary inertia, but assumed a non-linear temperature distribution across the shell thickness. The behavior of multilayered cylinder subjected to high rate of thermal loading was studied by Wang et al [23]. GaoC and Noda [4] presented clear investigations on the thermal-induced interfacial cracking of magneto electro elastic materials under uniform heat flow. The point temperature solution for a penney-shaped crack in an infinite transversely isotropic thermo-piezo-elastic medium subjected to a concentrated thermal load was analyzed by Chen et al [1]. Wang [22] studied the vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load. Martin and Berger [8] analyzed the propagation of waves in woods, especially free vibrations in a wooden pole. A three-dimensional vibration of a simply supported, homogeneous transversely isotropic coupled thermoelastic cylindrical panel was examined by Sharma [17]. Sharma and Sharma [19] extended the analysis [17] to study the vibration of transversely isotropic thermoelastic cylindrical panel in the context of generalized thermoelasticity. Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross section was studied by Ponnusamy [13]. Suhubi and Erbey [21] investigated longitudinal wave propagation in thermoelastic cylinder. Sharma et al. [18] presented the clear investigation of the vibrations of a thermoelastic cylindrical panel with voids. Selvamani and Ponnusamy [15] analysed the three-dimensional wave propagation in a homogeneous isotropic rotating cylindrical panel in the context of three dimensional linear theory of elasticity. Ponnusamy and Selvamani [14] extended the analysis [15] to study the flexure wave propagation in a homogeneous isotropic rotating cylindrical panel in the context of three dimensional linear theory of elasticity.

The objective of the present paper is to investigate the three dimensional vibrations in a simply supported, homogeneous transversely isotropic, rotating cylindrical panel. Three displacement potential functions are employed for solving the equation of motion and heat equation. The purely transverse waves get decoupled from the rest of motion and are not affected by thermal field. By using the method of separation of variables the model of instant vibration problem is reduced to a system of four second order coupled ordinary differential equations in radial coordinate. The secular equation which governs the three dimensional vibration of rotating cylindrical panel has been derived by using Matrix Frobenius method. The numerical solution of secular equation has been carried out by MATLAB programming to compute lowest frequency, frequency shift, phase velocity and thermoelastic damping factor which have been presented graphically with respect to the various parameter for first two modes of vibrations.

2. FORMULATION OF THE PROBLEM AND FORMAL SOLUTION

We consider a homogeneous transversely isotropic, thermal conducting elastic cylindrical panel of length L at uniform temperature T_0 in the undisturbed state, initially, with rotational speed Ω'. The inner and outer radius of the cylinder is given by a and b, respectively with thickness h. The basic governing equations of motion and heat conduction of three-dimensional linear coupled thermoelasticity for homogeneous, transversely isotropic, materials with rotation speed Ω' about z-axis in cylindrical co-ordinates (r,θ,z) system, in the absence of body forces and heat source, are given by [15]

$$\sigma_{rr,r} + \frac{1}{r}\sigma_{r\theta,\theta} + \sigma_{rz,z} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + \rho\Omega'^2 u_r = \rho\frac{\partial^2 u_r}{\partial t^2}$$

$$\sigma_{r\theta,r} + \frac{1}{r}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + \frac{2\sigma_{r\theta}}{r} + \rho\Omega'^2 u_\theta = \rho\frac{\partial^2 u_\theta}{\partial t^2} \tag{1}$$

$$\sigma_{rz,r} + \frac{1}{r}\sigma_{\theta z,\theta} + \sigma_{zz,z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 u_z}{\partial t^2}$$

$$\mathrm{K}_1\left(\mathrm{T}_{,rr} + r^{-1}\mathrm{T}_{,r} + r^{-2}\mathrm{T}_{,\theta\theta}\right) + \mathrm{K}_3\mathrm{T}_{,zz} - \rho C_e\dot{T} = T_0\left[\beta_1\left(\dot{e}_{rr}+\dot{e}_{\theta\theta}\right)+\beta_3\dot{e}_{zz}\right] \tag{2}$$

The stress-strain relations for homogeneous, transversely isotropic, thermo elastic material in cylindrical co-ordinate system is

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$

$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - \beta_1 T$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - \beta_3 T \tag{3}$$

$$\sigma_{r\theta} = 2c_{66}e_{r\theta},\quad \sigma_{\theta z} = 2c_{44}e_{\theta z},\quad \sigma_{rz} = 2c_{44}e_{rz}$$

The relation between strain and displacement are

$$e_{rr} = \frac{\partial u_r}{\partial r},\; e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r},\; e_{zz} = \frac{\partial u_z}{\partial z},\; e_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\theta}\right)$$

$$e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),\quad e_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \tag{4}$$

$$c_{66} = \frac{c_{11}-c_{12}}{2}\quad \beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3,\quad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3 \tag{5}$$

Here $\mathbf{u} = (u_r,\, u_\theta,\, u_z)$ is the displacement vector; $T(r,\theta,z,t)$ is the temperature change; $c_{11},\ c_{12},\ c_{13}, c_{33}$ and c_{44} are five elastic constants; $\alpha_1,\ \alpha_3$ and K_1, K_3 are the coefficients of linear thermal expansion and thermal conductivities along and perpendicular to the axis of symmetry respectively; ρ and C_e are the mass density and specific heat at constant strain respectively, e_{ij} the strain tensor ; σ_{ij} the stress tensor. The comma notation is used for spatial-derivatives and superimposed dot denotes time derivatives.

We consider the free vibrations of a right circular cylindrical panel with rotation speed Ω' subjected to traction free and thermally insulated boundary conditions on the surfaces $r = a, b$ and which is simply supported on the edges $z = 0$ and $z = L$.

Here we have used the following non-dimensional quantities

$$\xi = \frac{r}{R},\; Z = \frac{z}{L},\; \tau = \frac{v_2 t}{R},\; U_\xi = \frac{u_r}{R},\; U_\theta = \frac{u_\theta}{R},\; U_Z = \frac{u_z}{R},\; \Theta = \frac{\mathrm{T}}{\mathrm{T}_0}$$

$$\left(\tau_{\xi\xi},\tau_{\theta\theta},\tau_{ZZ}\right) = \frac{1}{c_{11}}\left(\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz}\right),\; \left(\tau_{\theta Z},\tau_{\xi Z},\tau_{\xi\theta}\right) = \frac{1}{c_{11}}\left(\sigma_{\theta z},\sigma_{rz},\sigma_{r\theta}\right)$$

$$R = \frac{a+b}{2}, c_1 = \frac{c_{33}}{c_{11}},\ c_2 = \frac{c_{44}}{c_{11}},\ c_3 = \frac{c_{13}+c_{44}}{c_{11}},\ c_4 = \frac{c_{66}}{c_{11}}$$

$$\overline{\mathrm{K}} = \frac{\mathrm{K}_3}{\mathrm{K}_1},\ \overline{\beta} = \frac{\beta_3}{\beta_1},\ \beta^* = \frac{\beta_1 \mathrm{T}_0}{c_{11}}$$

$$\varepsilon_1 = \frac{\beta_1^2 \mathrm{T}_0}{\rho C_e c_{11}}, A_R = \frac{R}{L}, \Omega^* = \frac{\omega^* R}{v_2} \tag{6}$$

where $\omega^* = \frac{C_e c_{11}}{\mathrm{K}_1}$, $v_2 = \sqrt{\frac{c_{44}}{\rho}}$ and $\Omega = \frac{\omega R}{v_2}$ are the characteristics frequency of vibrations, velocity of purely elastic shear wave in medium and the non dimensional circular frequency, respectively.

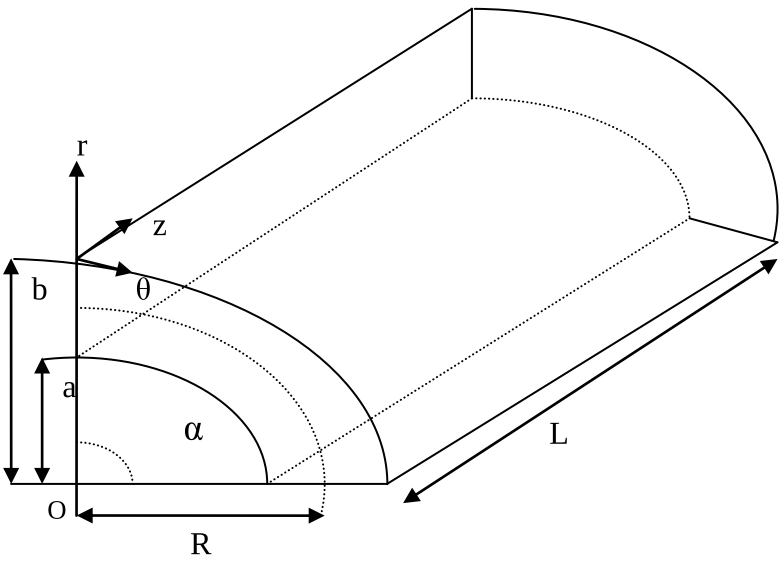

Fig.1. Geometry of problem

Upon using non-dimensionless quantities (6) in equations (1) to (2), we get

$$\left[\left(\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}-\frac{1}{\xi^2}\right)+c_4\frac{1}{\xi}\frac{\partial^2}{\partial\theta^2}-c_2\frac{\partial^2}{\partial\tau^2}\right]U_\xi$$
$$+\left[(1-c_4)\frac{1}{\xi}\frac{\partial^2}{\partial\xi\partial\theta}-(1+c_4)\frac{1}{\xi^2}\frac{\partial}{\partial\theta}\right]U_\theta+\left[c_3A_R\frac{\partial^2}{\partial\xi\partial Z}\right]U_Z-\beta^*\frac{\partial\Theta}{\partial\xi}=0 \tag{7.1}$$

$$\left[(1-c_4)\frac{1}{\xi}\frac{\partial^2}{\partial\xi\partial\theta}+(1+c_4)\frac{1}{\xi^2}\frac{\partial}{\partial\theta}\right]U_\xi$$
$$+\left[c_4\left(\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}-\frac{1}{\xi^2}\right)+\frac{1}{\xi^2}\frac{\partial^2}{\partial\theta^2}+c_2A_R^2\frac{\partial^2}{\partial Z^2}-c_2\frac{\partial^2}{\partial\tau^2}\right]U_\theta+c_3A_R\frac{1}{\xi}\frac{\partial^2U_Z}{\partial\theta\partial Z}-\beta^*\frac{1}{\xi}\frac{\partial\Theta}{\partial\theta}=0 \tag{7.2}$$

$$\left[c_3A_R\left(\frac{\partial^2}{\partial Z\partial\theta}+\frac{1}{\xi}\frac{\partial}{\partial Z}\right)\right]U_\xi+c_3A_R\frac{\partial^2U_\theta}{\partial\theta\partial Z}$$
$$+\left[c_2\left(\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}+\frac{1}{\xi^2}\right)+\frac{1}{\xi^2}\frac{\partial^2}{\partial\theta^2}+c_1A_R^2\frac{\partial^2}{\partial Z^2}-c_2\frac{\partial^2}{\partial\tau^2}\right]U_Z-A_R\bar{\beta}\beta^*\frac{\partial\Theta}{\partial Z}=0 \tag{7.3}$$

$$\left[\left(\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}+\frac{1}{\xi^2}\frac{\partial^2}{\partial\theta^2}\right)+\bar{\mathrm{K}}A_R^2\frac{\partial^2}{\partial Z^2}-c_2\Omega^*\frac{\partial}{\partial\tau}\right]\Theta$$
$$-\frac{c_2\Omega^*\varepsilon_1}{\beta^*}\frac{\partial}{\partial\tau}\left[\left(\frac{\partial}{\partial\xi}+\frac{1}{\xi}\right)U_\xi+\frac{1}{\xi}\frac{\partial U_\theta}{\partial\theta}+A_R\bar{\beta}\frac{\partial U_Z}{\partial Z}\right]=0 \tag{7.4}$$

In order to facilitate the solution, we introduce potential functions $\psi,\ G,\ W$ as used by Sharma [17]

$$U_\xi=\frac{1}{\xi}\frac{\partial\psi}{\partial\theta}-\frac{\partial G}{\partial\xi},\ \ U_\theta=-\frac{1}{\xi}\frac{\partial G}{\partial\theta}-\frac{\partial\psi}{\partial\xi},\ \ U_Z=A_R\frac{\partial W}{\partial Z} \tag{8}$$

Using equation (8) in equations (7.1)-(7.4) we find that G,W,ψ,Θ as used by satisfies the equations

$$\left(\nabla_1^2+c_2A_R^2\frac{\partial^2}{\partial Z^2}-c_2\frac{\partial^2}{\partial\tau^2}+\Gamma'\Omega'^2\right)G-c_3A_R^2\frac{\partial^2W}{\partial Z^2}+\beta^*\Theta=0 \tag{9.1}$$

$$-c_3\nabla_1^2G+c_2\left(\nabla_1^2+\frac{c_1}{c_2}A_R^2\frac{\partial^2}{\partial Z^2}-\frac{\partial^2}{\partial\tau^2}\right)W-\bar{\beta}\beta^*\Theta=0 \tag{9.2}$$

$$\left(\nabla_1^2+\bar{\mathrm{K}}A_R^2\frac{\partial^2}{\partial Z^2}-\frac{\omega^*c_2R}{v_2}\frac{\partial}{\partial\tau}\right)\Theta+\frac{c_2\Omega^*\varepsilon_1}{\beta^*}\frac{\partial}{\partial\tau}\left(\nabla_1^2G-\bar{\beta}A_R^2\frac{\partial^2W}{\partial Z^2}\right)=0 \tag{9.3}$$

$$\left(\nabla_1^2+\frac{c_2}{c_4}A_R^2\frac{\partial^2}{\partial Z^2}-\frac{c_2}{c_4}\frac{\partial^2}{\partial\tau^2}+\frac{\Gamma'}{c_4}\Omega'^2\right)\psi=0 \tag{9.4}$$

where

$$\nabla_1^2=\frac{\partial^2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial}{\partial\xi}+\frac{1}{\xi^2}\frac{\partial^2}{\partial\theta^2},\ \ \Gamma'=\frac{\rho R^2}{c_{11}} \tag{10}$$

The equation (9.4) represents purely transverse wave, which is not affected by the temperature change. This wave is polarized in planes perpendicular to z-axis. We consider the free vibrations of a cylindrical panel with simply supported edge and rotating with speed Ω'

subject to traction free and thermally insulated boundary conditions at the surfaces $r = a, b$. Therefore, we assume solution for three displacement functions and temperature change as

$$G(\xi,\theta,Z,\tau) = \overline{G}(\xi)\sin(m\pi Z)\exp\left\{i\left(\frac{n\pi}{\alpha}\theta - \Omega\tau\right)\right\}$$

$$W(\xi,\theta,Z,\tau) = \overline{W}(\xi)\sin(m\pi Z)\exp\left\{i\left(\frac{n\pi}{\alpha}\theta - \Omega\tau\right)\right\}$$

$$\Theta(\xi,\theta,Z,\tau) = \overline{\Theta}(\xi)\sin(m\pi Z)\exp\left\{i\left(\frac{n\pi}{\alpha}\theta - \Omega\tau\right)\right\}$$

$$\psi(\xi,\theta,Z,\tau) = \overline{\psi}(\xi)\sin(m\pi Z)\exp\left\{i\left(\frac{n\pi}{\alpha}\theta - \Omega\tau\right)\right\} \quad (11)$$

The solution are applicable to both closed hollow cylinder and open ones (panels), depending upon whether $\frac{n\pi}{\alpha}$ is an integer and not.

On using solutions (11) in equations (9), we get

$$\left(\nabla_2^2 + g_1\right)\overline{G} + g_2\overline{W} + \beta^*\overline{\Theta} = 0 \quad (12.1)$$

$$-c_3\nabla_2^2\overline{G} + c_2\left(\nabla_2^2 + g_3\right)\overline{W} - \overline{\beta}\,\beta^*\overline{\Theta} = 0 \quad (12.2)$$

$$g_5\left(\nabla_2^2\overline{G} + \overline{\beta}\,t_L^2\overline{W}\right) + \beta^*\left(\nabla_2^2 + g_4\right)\overline{\Theta} = 0 \quad (12.3)$$

$$(\nabla_2^2 + k_1^2)\overline{\psi} = 0 \quad (12.4)$$

where

$$\nabla_2^2 = \frac{d^2}{d\xi^2} + \frac{1}{\xi}\frac{d}{d\xi} - \frac{\beta^2}{\xi^2} \quad (13)$$

$$g_1 = c_2\left[\Omega^2\left(1 + v_2^2R^2\Gamma^2\right) - t_L^2\right],\ g_2 = c_3t_L^2,\ g_3 = \Omega^2 - \frac{c_1}{c_2}t_L^2$$

$$g_4 = \frac{ic_2}{\omega'}\left(\Omega^2 - \frac{i\overline{\mathrm{K}}\,t_L^2\,\omega'}{c_2}\right),\ g_5 = \frac{\varepsilon_1c_2\Omega^2}{i\omega'},\ k_1^2 = \frac{c_2}{c_4}\left[\Omega^2\left(1 + \frac{\rho R^2}{c_2}\Gamma^2\right) - t_L^2\right]$$

$$\omega' = \frac{\omega}{\omega^*},\ \Gamma = \frac{\Omega'}{\Omega}\ \ t_L = \frac{m\pi R}{L},\ \ \beta = \frac{n\pi}{\alpha} \quad (14)$$

Clearly in the transformed domain, the cylindrical panel encloses the region $\eta_1 \le r \le \eta_2$, $0 \le Z \le 1$ and $0 \le \theta \le \alpha$, where $\eta_1 = \frac{a}{R}$ and $\eta_2 = \frac{b}{R}$.

The equation (12.4) is a Bessel's equation and its possible solutions are

$$\overline{\psi}(r) = \begin{cases} E_7J_\beta(k_1r) + E_7'Y_\beta(k_1r), & k_1^2 > 0 \\ E_7r^\beta + E_7'r^{-\beta}, & k_1^2 = 0 \\ E_7I_\beta(k_1'r) + E_7'K_\beta(k_1'r), & k_1^2 < 0 \end{cases} \quad (15)$$

where $k_1'^2 = -k_1^2$. Here E_7 and E_7' are two arbitrary constants, and J_β and Y_β are Bessel functions for first and second kind and I_β and K_β are modified Bessel functions for the first and second kind respectively. Generally $k_1^2 \neq 0$, so we go on with our derivation by taking the form of $\overline{\psi}$ for $k_1^2 < 0$, the derivation for $k_1^2 > 0$ is obviously similar. Therefore the solution valid in case of cylindrical panel is taken here as

$$\overline{\psi}(r) = E_7 I_\beta(k_1' r) + E_7' K_\beta(k_1' r) \tag{16}$$

The equations (10.1)-(10.3) are system of coupled differential equations, which can be written in matrix form as

$$\mathbf{A}\nabla_2^2 \overline{\mathbf{Z}} = -\mathbf{B}\overline{\mathbf{Z}} \tag{17}$$

$$\text{where } \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ -c_3 & c_2 & 0 \\ g_5 & 0 & \beta^* \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} g_1 & g_2 & \beta^* \\ 0 & c_2 g_3 & -\overline{\beta}\beta^* \\ 0 & g_5 \overline{\beta} t_L^2 & \beta^* g_4 \end{pmatrix} \quad \overline{\mathbf{Z}} = \begin{pmatrix} \overline{G} \\ \overline{W} \\ \overline{\Theta} \end{pmatrix} \quad \mathbf{Z} = \begin{pmatrix} G \\ W \\ \Theta \end{pmatrix} \tag{18}$$

3. MATRIX FROBENIUS METHOD

A standard technique for solving ordinary differential equations is the method of Frobenius, in which the solutions are in the form of power series. Clearly $\xi = 0$ is a regular singular point of the matrix differential equations (14) and hence, we take the solution of type

$$\overline{\mathbf{Z}}(s,\xi) = \sum_{k=0}^{\infty} \mathbf{Z}_k(s)\xi^{s+k} \tag{19}$$

$$\text{where} \quad \overline{\mathbf{Z}}(s,\xi) = \begin{pmatrix} \overline{G}(s,\xi) \\ \overline{W}(s,\xi) \\ \overline{\Theta}(s,\xi) \end{pmatrix}, \quad \mathbf{Z}_k(s) = \begin{pmatrix} L_k(s) \\ M_k(s) \\ N_k(s) \end{pmatrix}.$$

Here the coefficients $L_k(s), M_k(s), N_k(s)$ and the parameter s (real or complex) are to be determined. On substituting expressions (19) in equations (17), we get

$$\mathbf{AZ_0}(s^2 - \beta^2)\xi^{s-2} + \mathbf{AZ_1}\left[(s+1)^2 - \beta^2\right]\xi^{s-1} + \sum_{k=0}^{\infty}\left[\mathbf{AZ_{k+2}}\left[(s+k+2)^2 - \beta^2\right] + \mathbf{BZ_k}\right]\xi^{s+k} = 0 \tag{20}$$

Equating to zero the coefficients of lowest power r^{s-2} in the resulting coupled differential equation, we obtain

$$\mathbf{A}(s^2 - \beta^2)\mathbf{Z_0} = 0 \tag{21}$$

The system of equations (21) will have non-trivial solution if and only if

$$\left|\mathbf{A}(s^2 - \beta^2)\right| = 0$$

This implies that $(s^2 - \beta^2)^3 |\mathbf{A}| = 0$ (22)

Because the matrix $\mathbf{A}$ is non singular, therefore the above condition (22) results in the following indicial equation

$$(s^2 - \beta^2)^3 = 0 \tag{23}$$

The roots of indicial equation (23) are given by

$$s_j = \begin{cases} \beta & j = 1,2,3 \\ -\beta & j = 4,5,6 \end{cases} \tag{24}$$

For this choice of the roots of indicial equation, the system of equations (19) leads to the eigenvector

$$\mathbf{Z_0} = \begin{bmatrix} 1 & \dfrac{c_3}{c_2} & \dfrac{-a}{\beta^*} \end{bmatrix}^T L_0 \tag{25}$$

Again equating to zero the coefficient of next lowest power ξ^{s-1}, we obtain

$$\mathbf{A}[(s+1)^2 - \beta^2]\mathbf{Z_1} = 0 \tag{26}$$

The choice of s_j $(j = 1$ *to* $6)$ given by equation (24) provides us

$$[(s+1)^2 - \beta^2]^3 |\mathbf{A}| \neq 0$$

So that $\mathbf{Z_1} = 0$ (27)

for each s_j $(j = 1 \; to \; 6)$. Now equating to zero the coefficient of ξ^{s+k}, we get the recurrence relations of $\mathbf{Z_k}(s_j)$ for $k \geq 2$ as

$$[(s+k+2)^2 - \beta^2]\mathbf{A}\,\mathbf{Z}_{k+2} + \mathbf{B}\,\mathbf{Z}_k = 0 \tag{28}$$

On simplification this provides us

$$\mathbf{Z_{k+2}} = \frac{1}{[(s+k+2)^2 - \beta^2]}\mathbf{C}\,\mathbf{Z_k}, \quad k \geq 2 \tag{29}$$

where the matrix $\mathbf{C}$ is given by

$$\mathbf{C} = -(\mathbf{A^{-1}B}) = \begin{pmatrix} -g_1 & -g_2 & -\beta^* \\ -\dfrac{g_1 c_3}{c_2} & -\dfrac{(c_3 g_2 + g_3 c_2)}{c_2} & \dfrac{(\overline{\beta} - c_3)\beta^*}{c_2} \\ \dfrac{g_5 g_1}{\beta^*} & \dfrac{g_5(g_2 - \overline{\beta} t_L^2)}{\beta^*} & (g_5 - g_4) \end{pmatrix} \tag{30}$$

The successive use of recurrence relations (29) along with equation (27) provides us

$$\mathbf{Z_{2k}}(s) = \frac{1}{[(s+2)^2 - \beta^2][(s+4)^2 - \beta^2]\cdots[(s+2k)^2 - \beta^2]} C^k \mathbf{Z_0}, \quad \forall k = 1, 2, \ldots \tag{31}$$

$$\mathbf{Z_{2k+1}} = 0, \qquad \forall \qquad k = 0, 1, 2, 3, \ldots \tag{32}$$

Thus the solution (19) becomes

$$\overline{\mathbf{Z}}(s,r) = \sum_{k=0}^{\infty} \mathbf{Z_{2k}}(s) r^{s+2k} \tag{33}$$

where $\mathbf{Z}_{2k}$ is defined in equation (31). The convergence of series (33) must of course consider.

4. CONVERGENCE ANALYSIS

According to Cullen [2], the matrix series $f(\mathbf{C}) = \sum_{k=0}^{\infty} a_k \mathbf{C}^k$, C being a square matrix is convergent if $\rho(\mathbf{C}) \leq \rho'$ where $f(z)$ is a function defined by a convergent power series

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \quad \text{for} |z| \leq \rho'$$

Here $\rho(C)$ is the spectral radius defined by $\rho(C) \le \max_i \sum_{j=1}^{m'} |a_{ij}|$, where $\mathbf{C} = (a_{ij})$ is a square matrix of order m', of the disk $D' = \{z \in \ell \ : \ |z| \le \rho(\mathbf{C})\}$ which encloses all the eigen values of the matrix $\mathbf{C}$ according to Gerschgorian circle theorem. Moreover, the function $f(\mathbf{C})$ being analytic, the series can be differentiated term by term and derived series also have same radius of convergence. For the square matrix $\mathbf{C}$ defined in the equation (27), we have

$$\rho(\mathbf{C}) \le \max_i \sum_{j=1}^{3} |a_{ij}| \tag{34}$$

where $\sum_{j=1}^{3} |a_{1j}| = |g_1 + g_2 + \beta^*|$,

$$\sum_{j=1}^{3} |a_{2j}| = \left| \frac{g_1 c_3 + g_2 c_3 + g_3 c_2 + \bar{\beta}\beta^* - c_3 \beta^*}{c_2} \right|$$

$$\sum_{j=1}^{3} |a_{3j}| = \left| \frac{a g_1 + a g_2 + a\beta^* - \beta^* g_4 - a\bar{\beta}\, t_l^2}{\beta^*} \right| \tag{35}$$

The equation (33) with the help of equation (31) and (32) can be rewritten as

$$\sum_{k=0}^{\infty} \mathbf{Z}_{2k}(s)\xi^{s+2k} = \mathbf{Z_0}\xi^s \sum_{k=0}^{\infty} a_{2k}\mathbf{C}^k$$

where $a_{2k} = \dfrac{r^{2k}}{[(s+2)^2 - \beta^2][(s+4)^2 - \beta^2]\cdots[(s+2k)^2 - \beta^2]}$

Thus, we have $\rho' = \lim_{k\to\infty} \left| \dfrac{a_{2k}}{a_{2k+2}} \right| = \lim_{k\to\infty} \left| \dfrac{[(s+2k+2)^2 - \beta^2]}{\xi^2} \right| = +\infty$

This shows that the power series converges in the entire plane and its radius of convergence is given by

$$\rho(C) \le \max_i \sum_{j=1}^{3} |a_{ij}| < \rho' \tag{36}$$

Using above facts, we note that the power series of matrix $\mathbf{C}$ is convergent for all finite and non-zero value of 'ξ'.

Case I: When the roots of indicial equation are distinct and differ by integer

The general solution of equation (19) has the form

$$\bar{Z}(\xi) = \sum_{i=1}^{6} E_i \bar{Z}_i(\beta, \xi) \tag{37}$$

where $\bar{Z}_i(s,\xi) = \dfrac{d^{i-1}}{ds^{i-1}} \left[\xi^s \sum_{k=0}^{\infty} Z_{2k}(s)\xi^{2k} \right]$; $i = 1,2,3,4,5,6$

and the quantities E_i ($i = 1 \text{ to } 6$) are arbitrary constants to be evaluated by using boundary condition. Hence, the functions $\bar{G}(\xi), \bar{W}(\xi), \bar{T}(\xi)$ can be written as

$$\{\bar{G}(\xi), \bar{W}(\xi), \bar{\Theta}(\xi)\} = \sum_{i=1}^{6} E_i \{\bar{G}_i(\beta,\xi), \bar{W}_i(\beta,\xi), \bar{\Theta}_i(\beta,\xi)\} \tag{38}$$

where

$$\overline{G}_i(s,\xi)=\frac{d^{i-1}}{ds^{i-1}}\left[\xi^s\sum_{k=0}^{\infty}L_{2k}(s)\xi^{2k}\right] \qquad ; \quad i=1,2,3,4,5,6$$

$$\overline{W}_i(s,\xi)=\frac{d^{i-1}}{ds^{i-1}}\left[\xi^s\sum_{k=0}^{\infty}M_{2k}(s)\xi^{2k}\right] \qquad ; \quad i=1,2,3,4,5,6$$

$$\overline{\Theta}_i(s,\xi)=\frac{d^{i-1}}{ds^{i-1}}\left[\xi^s\sum_{k=0}^{\infty}N_{2k}(s)\xi^{2k}\right] \qquad ; \quad i=1,2,3,4,5,6 \tag{39}$$

The potential functions G, W, Θ, ψ written from equations (11) by using equations (39) along with equation (18) as under

$$\{G(\xi), W(\xi), \Theta(\xi)\}=\left\{\sum_{i=1}^{6}E_i\{\overline{G}_i(\beta,\xi), \overline{W}_i(\beta,\xi), \overline{\Theta}_i(\beta,\xi)\}\right\}\sin(m\pi Z)\exp\{i\ (\beta\theta-\Omega\tau)\}$$

$$\psi(\xi,\theta,Z,\tau)=\{E_7 I_\beta(k_1'\xi)+E_7' K_\beta(k_1'\xi)\}\sin(m\pi Z)\exp\{i\,(\beta\theta-\Omega\tau)\} \tag{40}$$

Case II: When the roots of the indicial equation are distinct and do not differ by integer Thus the general solution of equation (19) has the form

$$\overline{Z}(\xi)=\sum_{i=1}^{3}E_i\overline{Z}_i(\beta,\xi)+\sum_{i=1}^{3}E_i'\overline{Z}_i(-\beta,\xi) \tag{41}$$

Here E_i' $(i=1,\ 2,\ 3)$ are arbitrary constants to be evaluated by using boundary conditions. Hence, the functions $\overline{G}(\xi), \overline{W}(\xi), \overline{\Theta}(\xi)$ can be written as

$$\{\overline{G}(\xi), \overline{W}(\xi), \overline{\Theta}(\xi)\}=\sum_{i=1}^{3}E_i\{\overline{G}_i(\beta,\xi), \overline{W}_i(\beta,\xi), \overline{\Theta}_i(\beta,\xi)\}+\sum_{i=1}^{3}E_i'\{\overline{G}_i(-\beta,\xi), \overline{W}_i(-\beta,\xi), \overline{\Theta}_i(-\beta,\xi)\} \tag{42}$$

The potential functions G, W, Θ, ψ written from equations (11) by using equations (42) along with equation (18) as under

$$\{G(\xi), W(\xi), \Theta(\xi)\}=$$

$$\left\{\sum_{i=1}^{3}E_i\{\overline{G}_i(\beta,\xi), \overline{W}_i(\beta,\xi), \overline{\Theta}_i(\beta,\xi)\}+\sum_{i=1}^{3}E_i'\{\overline{G}_i(-\beta,\xi), \overline{W}_i(-\beta,\xi), \overline{\Theta}_i(-\beta,\xi)\}\right\}\sin(m\pi Z)\exp\{i\ (\beta\theta-\Omega\tau)\}$$

$$\psi(\xi,\theta,Z,\tau)=\{E_7 I_\beta(k_1'\xi)+E_7' K_\beta(k_1'\xi)\}\sin(m\pi Z)\exp\{i\,(\beta\theta-\Omega\tau)\} \tag{43}$$

Thus we have derived a convergent formal solution of the model for both cases of the roots of indicial equation.

5. BOUNDARY CONDITIONS

We consider the free vibrations of a cylindrical panel which is subjected to two types of boundary condition at the lower and upper surfaces $(\xi=\eta_1,\eta_2)$

(a) *Mechanical condition:* The surfaces $(\xi=\eta_1,\eta_2)$ of panel have been assumed to be traction free, so that

$$\tau_{\xi\xi}=\tau_{\xi\theta}=\tau_{\xi Z}=0 \tag{44}$$

(b) *Thermal condition:* The surfaces $(\xi=\eta_1,\eta_2)$ of panel are either to be thermally insulated or isothermal. This leads to the conditions:

Insulated: $\Theta_{,\xi}=0$ (45.1)

Isothermal: $\Theta=0$ (45.2)

6. SECULAR EQUATIONS

In this section we shall derive secular equations for thermoelastic cylindrical panel, subjected to traction free, thermally insulated and traction free, isothermal at lower and upper surface $(\xi = \eta_1, \eta_2)$.

The displacement and stresses are obtained as

$$U_\xi = \left(-\overline{G}' + \frac{i\beta}{\xi}\overline{\psi} \right) \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$U_\theta = \left(-\frac{\beta}{\xi}\overline{G} - \overline{\psi}' \right) \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$U_Z = t_L \overline{W} \cos(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\} \tag{46}$$

$$\tau_{\xi\xi} = \left[g_1 \overline{G} + \frac{2c_4}{\xi}\overline{G}' - \frac{2c_4\beta^2}{\xi^2}\overline{G} - \frac{2c_4\beta}{\xi}\left(\overline{\psi}' - \frac{\overline{\psi}}{\xi} \right) + c_2 t_L^2 \overline{W} \right] \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$\tau_{\xi\theta} = c_4 \left[-\frac{2\beta}{\xi}\overline{G}' + \frac{2\beta}{\xi^2}\overline{G} - \overline{\psi}'' - \frac{\beta^2}{\xi^2}\overline{\psi} + \frac{\overline{\psi}'}{\xi} \right] \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$\tau_{\xi Z} = c_2\, t_L \left[-\overline{G}' - \frac{\beta}{\xi}\overline{\psi} + \overline{W}' \right] \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\} \tag{47}$$

Case I: Upon using the equations (38) and (16), with the help of equations (40), (46) and (47), the stresses and temperature gradient are obtained as

$$\tau_{\xi\xi} = \left\{ \sum_{i=1}^{6} E_i F_i + E_7 F_7 + E_7' F_7' \right\} \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$\tau_{\xi\theta} = \left\{ \sum_{i=1}^{6} E_i F_i^{*} + E_7 F_7 + E_7' {F_7^{*}}' \right\} \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$\tau_{\xi Z} = \left\{ \sum_{i=1}^{6} E_i F_i^{**} + E_7 F_7 + E_7' {F_7^{**}}' \right\} \cos(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\}$$

$$\Theta_{,\xi} = \sum_{i=1}^{6} \{E_i F_i^{***}\} \sin(m\pi Z) \exp\{i\ (\beta\theta - \Omega\tau)\} \tag{48}$$

where

$$F_i = \left[\frac{2c_4}{\xi}\overline{G}_i'(\beta,\xi) + \left(g_1 - \frac{2c_4\beta^2}{\xi^2} \right)\overline{G}_i(\beta,\xi) + c_2 t_L^2 \overline{W}_i(\beta,\xi) \right]$$

$$F_7 = \left[-\frac{2c_4\beta k_1'}{\xi} I_\beta'(k_1'\xi) \right] + \left[\frac{2c_4\beta}{\xi^2} I_\beta(k_1'\xi) \right]$$

$$F_i^{*} = \left[-\frac{2\beta}{\xi}\overline{G}_i'(\beta,\xi) + \frac{2\beta}{\xi^2}\overline{G}_i(\beta,\xi) \right] \quad F_7^{*} = \left[-k_1'^2 I_\beta''(k_1'\xi) + \frac{k_1'}{\xi} I_\beta'(k_1'\xi) - \frac{\beta^2}{\xi^2} I_\beta(k_1'\xi) \right]$$

$$F_i^{**} = \left[-\overline{G}_i'(\beta,\xi) + \overline{W}_i'(\beta,\xi) \right], \quad F_7^{**} = \left[\frac{\beta}{\xi^2} I_\beta(k_1'\xi) \right]$$

$$F_i^{***} = \overline{\Theta}_i'(\beta,\xi) \tag{49}$$

Employing the boundary conditions (44) and (45), we again get a systems of eight simultaneous equations in $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_7'$, which will have a non-trivial solution if the determinant of coefficients vanishes. This requirement of non trivial solution leads to secular equation

$$\left| p_{ij} \right| = 0 \qquad \forall\, i, j = 1, 2, 3, 4, 5, 6, 7, 8 \qquad (50)$$

$$p_{1j} = \frac{2c_4}{\eta_1} \bar{G}_j'(\beta, \eta_1) + \left(g_1 - \frac{2c_4 \beta^2}{\eta_1^2} \right) \bar{G}_j(\beta, \eta_1) + c_2 t_L^2 \bar{W}_j(\beta, \eta_1) \quad \forall\ j = 1, 2, 3, 4, 5, 6$$

$$p_{17} = -\frac{2c_4 \beta k_1'}{\eta_1} I_\beta'(k_1' \eta_1) + \frac{2c_4 \beta}{\eta_1^2} I_\beta(k_1' \eta_1)$$

$$p_{18} = -\frac{2c_4 \beta k_1'}{\eta_1} K_\beta'(k_1' \eta_1) + \frac{2c_4 \beta}{\eta_1^2} K_\beta(k_1' \eta_1)$$

$$p_{2j} = -\frac{2\beta}{\eta_1} \bar{G}_j'(\beta, \eta_1) + \frac{2\beta}{\eta_1^2} \bar{G}_j(\beta, \eta_1) \qquad \forall\ j = 1, 2, 3, 4, 5, 6$$

$$p_{27} = -k_1'^2 I_\beta''(k_1' \eta_1) + \frac{k_1'}{\eta_1} I_\beta'(k_1' \eta_1) - \frac{\beta^2}{\eta_1^2} I_\beta(k_1' \eta_1)$$

$$p_{28} = -k_1'^2 K_\beta''(k_1' \eta_1) + \frac{k_1'}{\eta_1} K_\beta'(k_1' \eta_1) - \frac{\beta^2}{\eta_1^2} K_\beta(k_1' \eta_1)$$

$$p_{3j} = -\bar{G}_j'(\beta, \eta_1) + \bar{W}_j'(\beta, \eta_1) \qquad \forall\ j = 1, 2, 3, 4, 5, 6$$

$$p_{37} = \frac{\beta}{\eta_1} I_\beta(k_1' \eta_1), \quad p_{38} = \frac{\beta}{\eta_1} K_\beta(k_1' \eta_1)$$

Insulated:

$$p_{4j} = \bar{\Theta}_j'(\beta, \eta_1) \qquad \forall\ j = 1, 2, 3, 4, 5, 6$$

$$p_{47} = 0 \ , \ p_{48} = 0$$

Isothermal

$$p_{4j} = \bar{\Theta}_j(\beta, \eta_1) \qquad \forall\ j = 1, 2, 3, 4, 5, 6 \qquad (51)$$

while p_{ij} , $i = 5, 6, 7, 8$ can be obtained by just replacing η_1 in p_{ij} by η_2 .

Here $\eta_1 = \frac{a}{R} = 1 - \frac{q}{2}$, $\eta_2 = \frac{b}{R} = 1 + \frac{q}{2}$, $q = \frac{(b-a)}{R}$

is the thickness to mean radius ratio of the cylindrical panel.

Case II: Upon using the equations (41) and (16), with the help of equations (43), (46) and (47), the stresses and temperature gradient are obtained as

$$\tau_{\xi\xi} = \left\{ \sum_{i=1}^{3} \{ E_i F_i + E_i' F_i' \} + E_7 F_7 + E_7' F_7' \right\} \sin(m\pi Z) \exp\{ i\ (\beta\theta - \Omega\tau) \}$$

$$\tau_{\xi\theta} = \left\{ \sum_{i=1}^{3} \left\{ E_i F_i^* + E_i' F_i^{*\prime} \right\} + E_7 F_7 + E_7' F_7^{*\prime} \right\} \sin(m\pi Z) \exp\{ i\ (\beta\theta - \Omega\tau) \}$$

$$\tau_{\xi Z} = \left\{ \sum_{i=1}^{3} \left\{ E_i F_i^{**} + E_i' F_i^{**\prime} \right\} + E_7 F_7 + E_7' F_7^{**\prime} \right\} \cos(m\pi Z) \exp\{ i\ (\beta\theta - \Omega\tau) \}$$

$$\Theta_{,\xi} = \sum_{i=1}^{6} \left\{ E_i F_i^{***} + E_i F_i^{***'} \right\} \sin(m\pi Z) \exp\{ i\ (\beta\theta - \Omega\tau) \} \tag{52}$$

$$F_i' = \left[\frac{2c_4}{\xi} \overline{G}_i'(-\beta, \xi) + \left(g_1 - \frac{2c_4\beta^2}{\xi^2} \right) \overline{G}_i(-\beta, \xi) + c_2 t_L^2 \overline{W}_i(-\beta, \xi) \right]$$

$$F_7' = \left[-\frac{2c_4 n\beta\, k_1'}{\xi} K_\beta'(k_1'\xi) \right] + \left[\frac{2c_4\beta}{\xi^2} K_\beta(k_1'\xi) \right]$$

$$F_i^{*'} = \left[-\frac{2\beta}{\xi} \overline{G}_i'(-\beta, \xi) + \frac{2\beta}{\xi^2} \overline{G}_i(-\beta, \xi) \right] F_7^{*'} = \left[-k_1'^2 K_\beta''(k_1'\xi) + \frac{k_1'}{\xi} K_\beta'(k_1'\xi) - \frac{\beta^2}{\xi^2} K_\beta(k_1'\xi) \right]$$

$$F_i^{**'} = \left[-\overline{G}_i'(-\beta, \xi) + \overline{W}_i'(-\beta, \xi) \right], F_7^{**'} = \left[\frac{\beta}{\xi^2} K_\beta(k_1'\xi) \right]$$

$$F_i^{***'} = \overline{\Theta}_i'(-\beta, \xi) \tag{53}$$

Upon employing the boundary conditions (44) and (45), we obtain systems of eight simultaneously equations in unknowns $E_1, E_1', E_2, E_2', E_3, E_3', E_7, E_7'$, which will provide us a non-trivial solution if the determinant of their coefficients vanishes. This requirement of non trivial solution leads to secular equation for cylindrical panel as

$$\left| p_{ij}' \right| = 0 \quad \forall i,\ j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8 \tag{54}$$

$$p_{1j}' = p_{1j} \qquad \forall\ j = 1,\ 2,\ 3, 7, 8$$

$$p_{1j}' = \frac{2c_4}{\eta_1} \overline{G}_j'(-\beta, \eta_1) + \left(g_1 - \frac{2c_4\beta^2}{\eta_1^2} \right) \overline{G}_j(-\beta, \eta_1) + c_2 t_L^2 \overline{W}_j(-\beta, \eta_1) \ \forall\ j = 4,\ 5,\ 6$$

$$p_{2j}' = p_{2j} \qquad \forall\ j = 1,\ 2,\ 3, 7, 8$$

$$p_{2j}' = -\frac{2\beta}{\eta_1} \overline{G}_j'(-\beta, \eta_1) + \frac{2\beta}{\eta_1^2} \overline{G}_j(-\beta, \eta_1) \qquad \forall\ j = 4,\ 5,\ 6$$

$$p_{3j}' = p_{3j} \qquad \forall\ j = 1,\ 2,\ 3, 7, 8$$

$$p_{3j}' = -\overline{G}_j'(-\beta, \eta_1) + \overline{W}_j'(-\beta, \eta_1) \qquad \forall\ j = 4,\ 5,\ 6$$

$$p_{4j}' = p_{4j} \qquad \forall\ j = 1,\ 2,\ 3, 7, 8$$

Insulated:

$$p_{4j}' = \overline{\Theta}_j'(-\beta, \eta_1) \qquad \forall\ j = 4,\ 5,\ 6$$

Isothermal:

$$p_{4j} = \overline{\Theta}_j(\beta, \eta_1) \qquad \forall\ j = 4,\ 5,\ 6 \tag{55}$$

while p_{ij}', $i = 5, 6, 7, 8$ can be obtained by just replacing η_1 in p_{ij}' by η_2.

7. SPECIAL CASES

In this section we shall discuss following particular cases of the secular equation in case of axisymmetric thermoelastic cylinder, transversely isotropic elastic cylinder and isotropic elastic cylinder and isotropic thermoelastic cylinder.

7.1 Axisymmetric thermoelastic cylinder

The analysis in case of an axisymmetric cylinder can be obtained by setting $\beta = 0$ in the present study throughout.

7.2 Homogeneous isotropic thermoelastic cylinder

In case of homogeneous isotropic material cylinder, we have

$$c_1 = 1, \quad c_2 = \frac{\mu}{\lambda + \mu}, \quad c_3 = \frac{\lambda + \mu}{\lambda + 2\mu}, \quad \bar{\beta} = 1, \quad \bar{K} = 1 \tag{56}$$

where λ , μ being the Lame's parameters. The analysis in the case of both axisymmetric cylinder and non-axisymmetric cylinder can be obtained from the present one by using relation (56) throughout.

7.3 Uncoupled thermoelastic cylinder

The analysis in case of uncoupled thermoelasticity can be obtained by taking $\varepsilon_1 = 0$ throughout in present one.

8. NON-ROTATING CYLINDRICAL PANEL

The analysis in the case of non-rotating thermo elastic or elastic cylindrical panel can be obtained by setting rotational speed $\Omega' = 0$ throughout the present one.

9. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate and verify results obtained in previous sections, we present some numerical simulation results. For the purpose of numerical computation, we considered the closed circular cylindrical shell with central angle $\alpha = 2\pi$ and integer n must be even since the shell vibrates in full circumferential wave. We have considered zinc-crystal like material whose physical data is given below (Dhaliwal and Singh [3]).

$c_{11} = 1.628 \times 10^{11}\, Nm^{-2}$, $c_{12} = 0.362 \times 10^{11}\, Nm^{-2}$, $c_{13} = 0.508 \times 10^{11}\, Nm^{-2}$

$c_{33} = 0.627 \times 10^{11}\, Nm^{-2}$, $c_{44} = 0.770 \times 10^{11}\, Nm^{-2}$, $\beta_1 = 5.75 \times 10^6\ Nm^{-2}\deg^{-1}$

$\beta_3 = 5.75 \times 10^6\ Nm^{-2}\deg^{-1}$, $K_1 = 1.24 \times 10^2\ Wm^{-2}\deg^{-1}$, $K_3 = 1.24 \times 10^2\ Wm^{-2}\deg^{-1}$

$\omega^{\bullet} = 5.01 \times 10^{11}\, s^{-1}, \rho = 7.14 \times 10^3 kgm^{-3}$, $T_0 = 296K$

The frequency equation for closed cylindrical shell can be obtained by setting $(\beta = l = 1,2,3\ldots)$, where β circumferential wave number. Due to presence of dissipation term in heat conduction equation, the frequency equation in general complex transcendental equation provides us complex value of frequency (Ω). For fixed value of β and k, the lowest frequency (Ω_R) and dissipation factor (D) are defined as

$$\Omega_R = \omega_R R\left(\frac{\rho}{c_{44}}\right)^{\frac{1}{2}}, \quad D = \omega_I R\left(\frac{\rho}{c_{44}}\right)^{\frac{1}{2}}$$

where $\omega_R = \mathrm{Re}(\omega)$ and $\omega_I = \mathrm{Im}(\omega)$. The phase velocity (c_{PH}) and frequency shift $(\Delta\omega_R)$ due to rotational are defined as:

$$c_{PH} = \frac{\Omega_R}{t_L}$$

$$\Delta\omega_R = \left| \frac{\omega_R(\Omega') - \omega_R(0)}{\omega_R(0)} \right| \tag{57}$$

where $\omega_R(\Omega')$ is the frequency in thermoelastic cylindrical shell with rotational speed Ω'. The thermoelastic damping factor is given by

$$Q^{-1} = 2\left|\frac{\text{Im}(\omega)}{\text{Re}(\omega)}\right| = 2\left|\frac{\omega_I}{\omega_R}\right| \tag{58}$$

The numerical computation has been carried out for $\beta > 0$, $k > 0$ with the help of MATLAB files .The secular equations (50) and (54) has been expressed in the form of $\Omega = g(\Omega)$ and the fixed point iteration numerical technique as outlined in Sharma [16] has been used to find approximate solution of $\Omega = g(\Omega)$ near to the initial guess of the root with tolerance (10^{-5}). The variations of computer simulated Lowest frequency, phase shift, frequency shift and thermoelastic damping factor of fixed value of circumferential wave number $(\beta = 1, 2)$, have been plotted versus different parameters in Figs 2 to 8.

Figs 2 and 3 represent the variations of phase velocity c_{PH} of first two modes of vibrations $(\beta = 1, 2)$, versus parameter Γ of the simply supported cylindrical shell of zinc-crystal like material, respectively. It is observed that the phase velocity increases monotonically with parameter Γ for $\Gamma < 1$ to attain maximum value at $\Gamma = 1$ and the decreases for $\Gamma < 1$ in both first and second modes of vibrations. Moreover, it has been noticed that the magnitude of phase shift to be larger in second mode of vibrations as compared to that of first mode of vibrations. Fig 4 shows the variations of lowest frequency versus rotational speed (Ω') of simply supported cylindrical shell of zinc-crystal like material for $(A_R = 10)$ of the cylindrical shell in case of two different modes of vibrations $(\beta = 1, 2)$. It is observed that the lowest frequency for each considered modes of vibrations $(\beta = 1, 2)$ increases monotonically with increasing the values of rotational speed (Ω') which is attributed to increase of coupling effect of various interacting fields due to increase of rotation of cylindrical shell. The magnitude of lowest frequency in case of second mode of vibrations is noticed to be large as compared to that of the first mode of vibrations. Figs 5 and 6 represent the variations of frequency shift due to rotation $(\Delta\omega_R)$ for each considered modes of vibrations $(\beta = 1, 2)$, versus axial wave number t_L for different values of rotational speed $(\Omega' = 0.2, 0.4, 0.6, 0.8)$ of the simply supported cylindrical shell, respectively. It is observed that the magnitude of frequency shift due to rotation $(\Delta\omega_R)$ increases to attain extreme values at lower value of axial wave number t_L and then decline to become steady and uniform at higher values of axial wave number t_L.

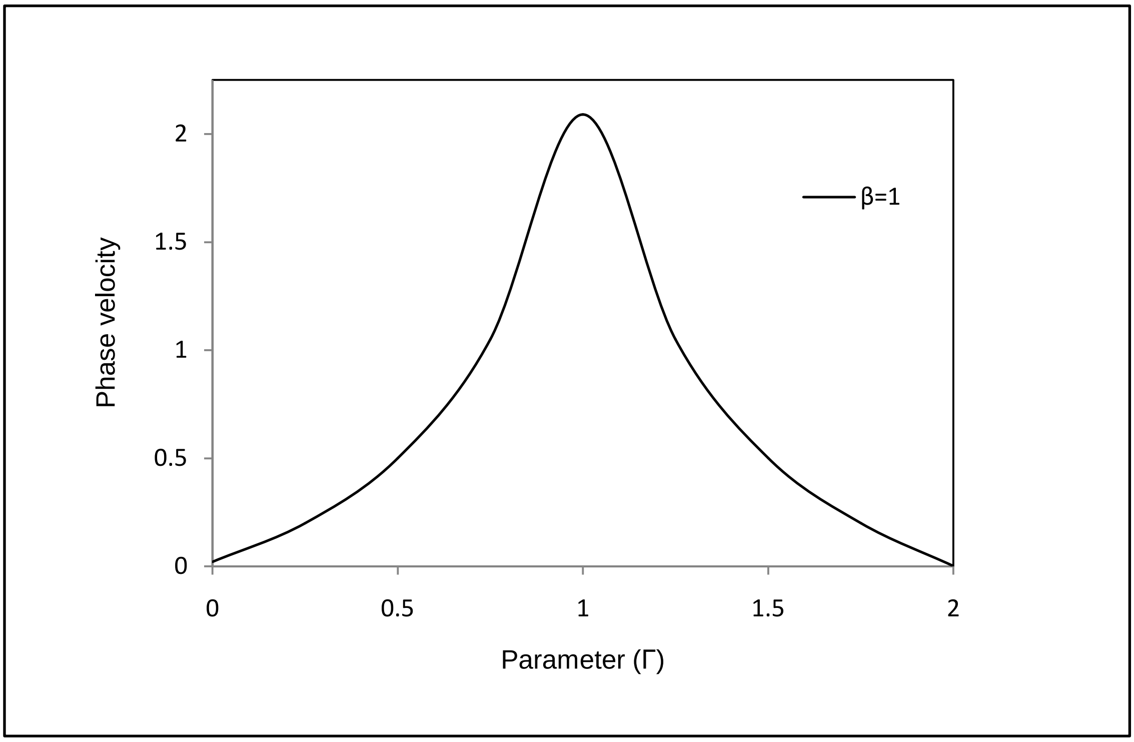

Fig.2. Phase velocity (c_{PH}) of circumferential wave number $(\beta = 1)$ versus parameter (Γ).

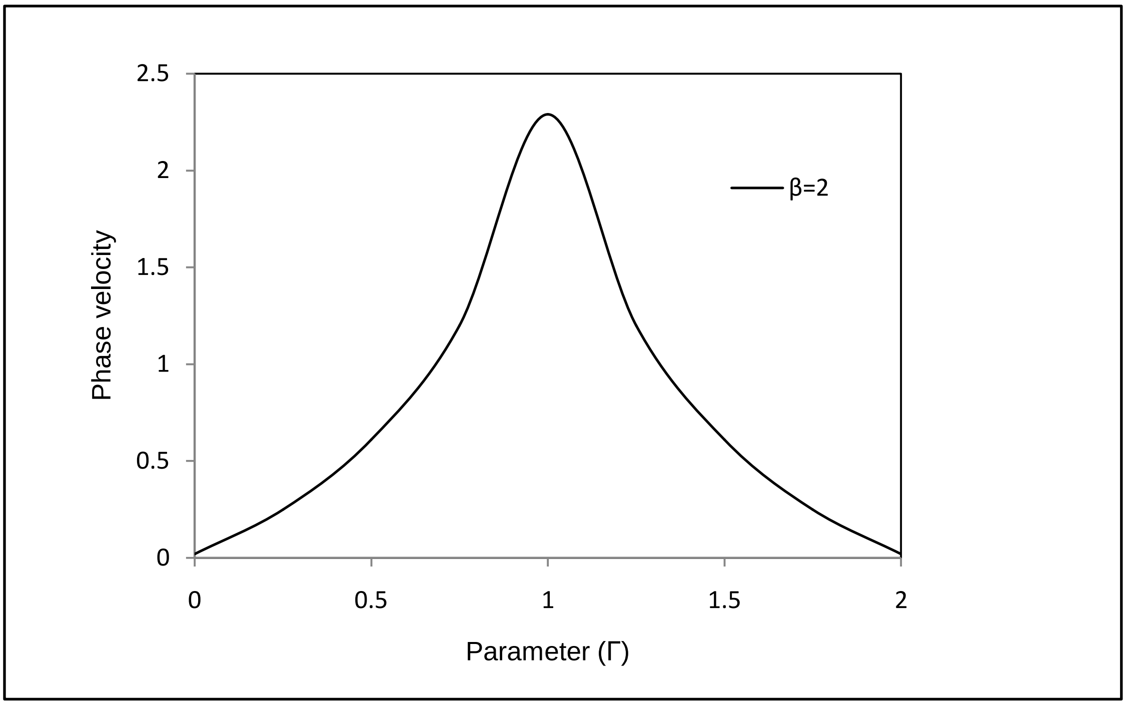

Fig. 3. Phase velocity (c_{PH}) of circumferential wave number $(\beta = 2)$ versus parameter (Γ).

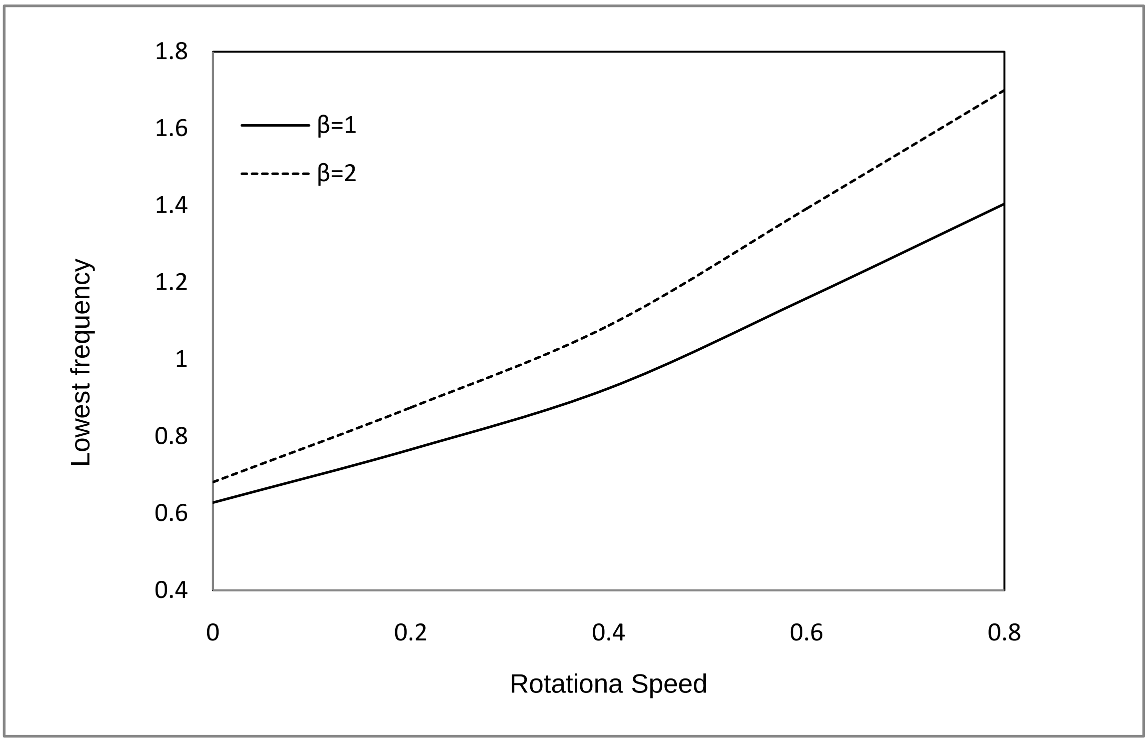

Fig. 4. Lowest frequency (Ω_R) of circumferential wave number $(\beta = 1, 2)$ versus rotational speed (Ω').

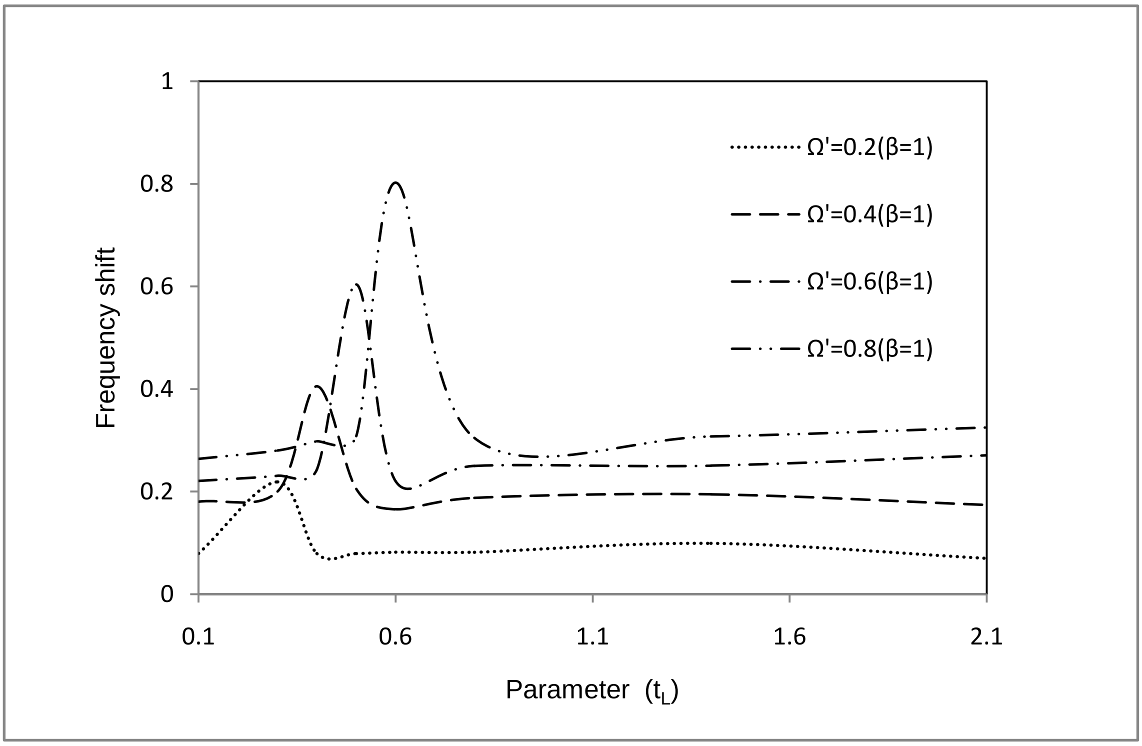

Fig. 5 Frequency shift $(\Delta\omega_R)$ of circumferential wave number $(\beta = 1)$ versus parameter (t_L) for different values of rotational speed Ω'.

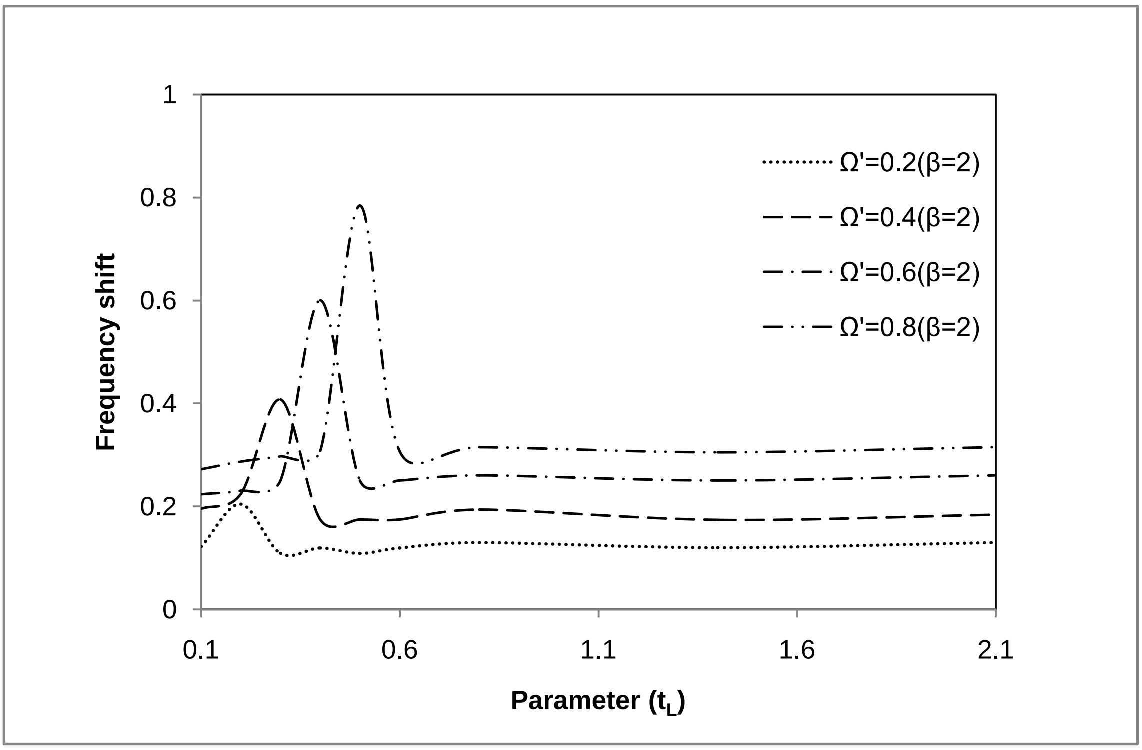

Fig. 6 Frequency shift $(\Delta\omega_R)$ of circumferential wave number $(\beta = 2)$ versus parameter (t_L) for different values of rotational speed Ω'

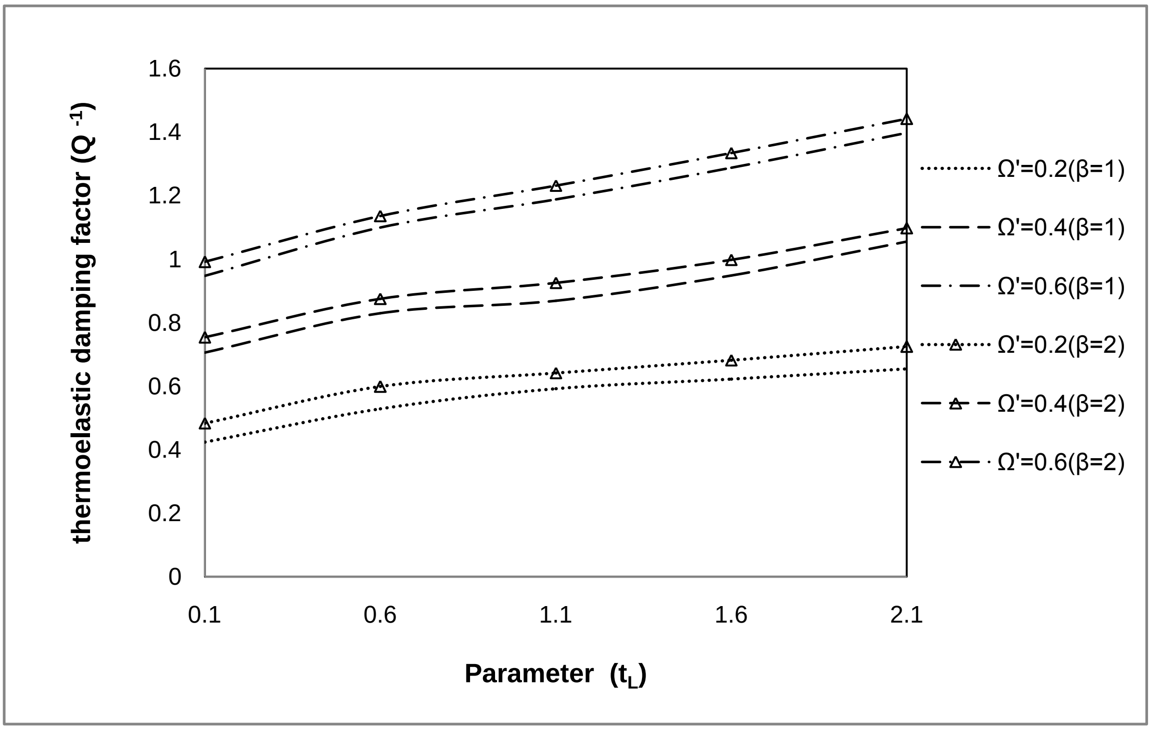

Fig. 7 Thermoelastic damping factor (Q^{-1}) of circumferential wave number $(\beta = 1, 2)$ versus parameter (t_L) for different values of rotational speed Ω'.

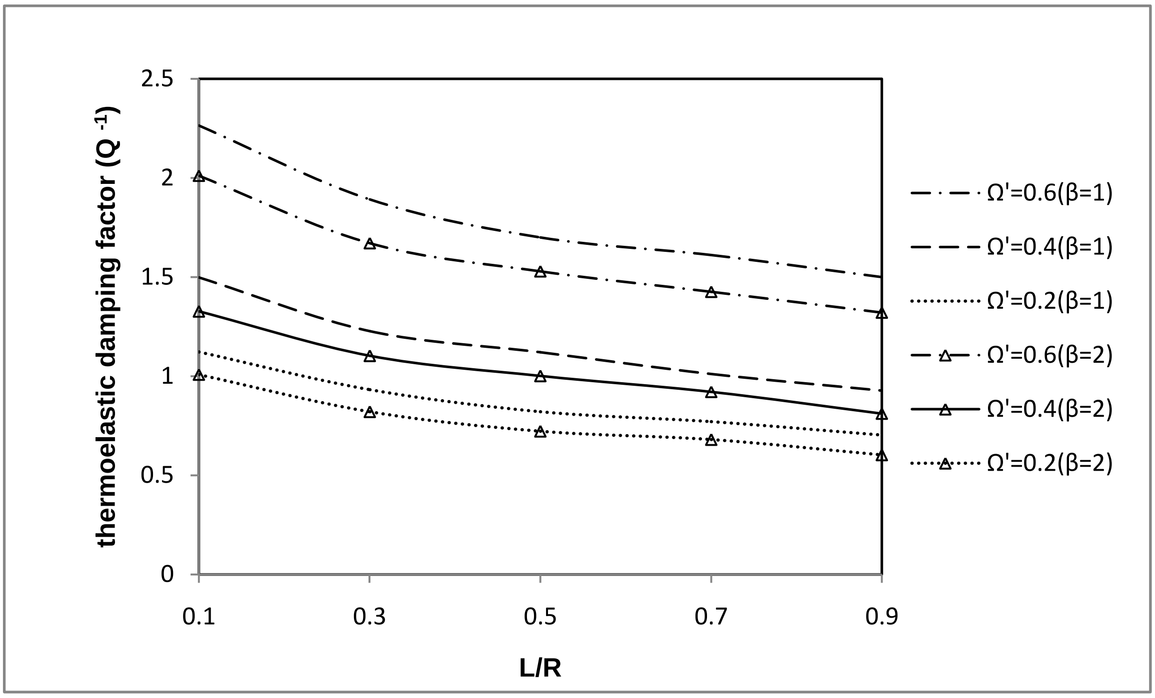

Fig. 8 Thermoelastic damping factor $\left(Q^{-1}\right)$ of circumferential wave number $(\beta = 1, 2)$ versus length to mean radius ratio (L/R) for different values of rotational speed Ω'.

Fig. 7 shows the variations of the thermoelastic damping factor $\left(Q^{-1}\right)$ of each mode of vibrations $(\beta = 1, 2)$ versus axial wave number t_L for different values of rotational speed $(\Omega' = 0.2, 0.4, 0.6, 0.8)$. It is observed that the thermoelastic damping factor $\left(Q^{-1}\right)$ of each mode of vibrations $(\beta = 1, 2)$ increases monotonically with the increasing values of axial wave number t_L for different rotational speed $(\Omega' = 0.2, 0.4, 0.6, 0.8)$ of the simply supported cylindrical shell. It is also noticed that the magnitude of thermoelastic damping factor $\left(Q^{-1}\right)$ of higher mode is larger as compared to that of smaller mode. Moreover, the magnitude of thermoelastic damping factor $\left(Q^{-1}\right)$ for each mode of vibrations $(\beta = 1,2)$ increases monotonically with the increasing value of the rotation speed Ω' of cylindrical shell. Fig. 8 displays the variations of the thermoelastic damping factor $\left(Q^{-1}\right)$ of each mode of vibrations $(\beta = 1, 2)$ versus length to mean radius ratio (L/R) for different values of rotational speed $(\Omega' = 0.2, 0.4, 0.6, 0.8)$. It is observed that the magnitude of thermoelastic damping factor $\left(Q^{-1}\right)$ of each mode of vibrations $(\beta = 1, 2)$ decreases monotonically with the increasing the value of length to mean radius ratio (L/R) and increases monotonically with the increasing value of the rotation speed Ω' of cylindrical shell. It is also observed that the magnitude of thermoelastic damping factor $\left(Q^{-1}\right)$ in case of first mode of vibrations is large as compared to that of the second mode of vibrations. Table 1 represents the variations of lowest frequency versus axial wave number (t_L) in simply supported cylindrical shell of zinc-crystal like material for rotational speed $(\Omega' = 0.2)$ for two modes of vibrations $(\beta = 1, 2)$. It is noticed that the lowest frequency for each considered mode of vibrations $(\beta = 1,2)$ increases monotonically with axial wave number (t_L). Moreover, it has been noticed that the magnitude of lowest frequency of second mode of vibrations is noticed to be large as compared to that of the first mode.

Table 1 The frequency parameter Ω of a thermoelastic cylindrical shell versus axial wave number t_L for $(\Omega' = 0.2)$ and $(\beta = 1,\ 2)$.

t_L / β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.0223	0.1001	0.3016	0.3897	0.5819	0.7115	0.8417	0.8917	1.1314	1.2657
2	0.1851	0.2899	0.4842	0.5391	0.6816	0.8701	0.9216	0.9816	1.2140	1.3506

Table 2 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ for a simply supported cylindrical shell for $m = 1,\ \nu = 0.3$ and $A_R = 20$.

$\frac{h}{R}$	β	**Reference[25]**	**Present**
0.05	1	0.0161	0.0162
	2	0.0393	0.0394
0.01	1	0.0161	0.0162
	2	0.0093	0.0094
0.002	1	0.0161	0.0162
	2	0.0054	0.0055

In order to check the validity of present analysis, we have used MATLAB codes to compute the non-dimensional frequency parameters $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ for non-rotating elastic simply-supported cylindrical shell made of steel with mass density $\rho = 7850 kgm^{-3}$, the Poisson's ratio $\nu = 0.3$ and Young's modulus $E = 2.1\times 10^{11} Nm^{-2}$ with thickness to mean radius ratio $\left(\frac{h}{R} = 0.002,\ 0.01,\ 0.05\right)$ and $\left(m = 1,\ \nu = 0.3,\ \frac{L}{R} = 20\right)$ for selected mode $(\beta = 1,2)$. The obtained results have been compared with those available in literature [25]. The computed results have been presented in Table 2, which exhibit excellent agreement with lowest frequency given in Table 1 and 3 of Ref [25]. Moreover, the computation of non-dimensional frequency (Ω) in a rotating and simply-supported elastic cylindrical shell made of copper material, have also been carried out for rotational speed $\Omega' = 0.4, 0.6, 0.8$ in case of $\beta = 1, 2$ and $t_L = 1$. The material properties of copper are given as [15]:

$\rho = 8.96 \times 10^3 kgm^{-3}$, $\lambda = 8.20 \times 10^{11} kgms^{-2}$, $\mu = 4.20 \times 10^{11} kgms^{-2}$, $\nu = 0.3$

$E = 2.139 \times 10^{11} Nm^{-2}$

The computed results have been presented in Table 3, which reveal a quite fair agreement with the natural frequency estimation plotted in Fig.1 of Ref [15].

Table 3 The frequency parameter $\Omega = \omega R \sqrt{(1-\nu^2)\rho / E}$ for a simply supported isotropic rotating elastic cylindrical shell $(t_L = 1)$.

Rotational speed (Ω')	β	Lowest frequency (Ω)
0.4	1	1.5984
	2	2.0913
0.6	1	1.6108
	2	3.0713
0.8	1	2.1014
	2	4.2013

10. CONCLUSIONS

The matrix Frobenius method in conjunction with modified Bessel functions have been successfully employed to study the vibrations of a homogeneous, transversely isotropic cylindrical panel based on three-dimensional thermoelasticity after decoupling the basic governing equations of motion and heat conduction with the use of potential functions to obtain the formal solution of simply supported rotating cylindrical panel. The convergence analysis of matrix Frobenius method has been successfully carried out and proved. The decoupled purely transverse mode is found to be independent of rest of the motion and temperature change. The various thermal, rotational and mechanical parameters have significant effects on the natural frequency, thermoelastic damping factor, and frequency shift of the cylindrical panel and results are presented as dispersion curves. The thermoelastic damping factor increases monotonically with axial wave number but decreases with length to mean radius ratio of the cylindrical panel. The variation of lowest frequency has been found to increase monotonically with rotational speed which is attributed to dissipation of energy, thermo-mechanical coupling and random behavior of molecules due to thermal and rotational variations. The magnitude of frequency shift attains extreme values at lower value of axial wave number and falls down to become steady and uniform at higher value of axial wave number. Resonance phenomenon has been found to occur because of increase of energy in vibrations which is attributed to random behavior of molecules due rotational variations. The present method may be robust and has computational suitability in addition to rapid convergence as compared to other methods available in literature.

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